Imperial College London

BIOE50011 – Signals and ***Control***

MATLAB practical 5 – State-space models

Learning objective

By the end of this MATLAB session you should be able to:

- Comfortably analyse and simulate an LTI system described by a state-space representation.
- Obtain the impulse response of a system described by a state-space representation.
- Simulate the time evolution of a system under a given time-dependent input.
- Design a state feedback controller for pole placement.
- Draw bode plots for LTI systems described in a state-space form.

Task 1 – Time response

The **unforced response** of the system

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \overbrace{\begin{bmatrix}-0.5 & -0.8\\ 0.8 & 0\end{bmatrix}}^{\boldsymbol{A}}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \overbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}^{\boldsymbol{B}}u,$$

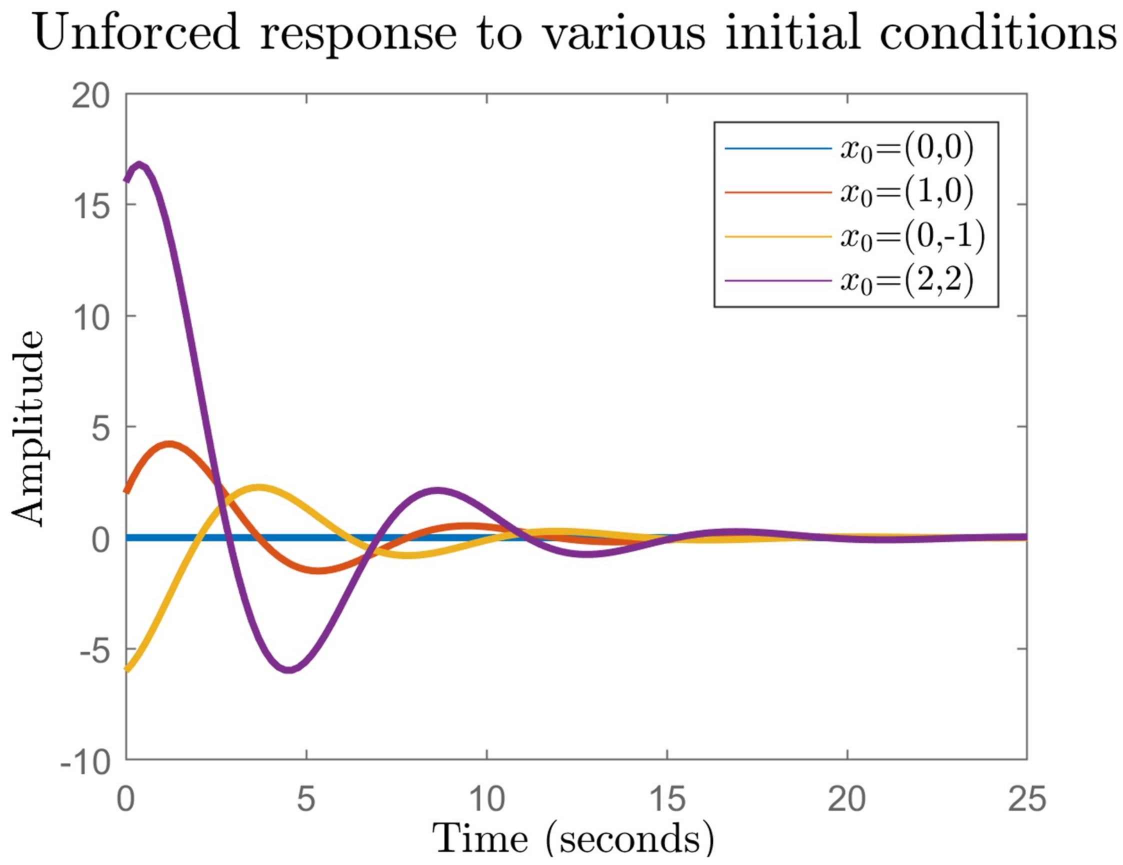
$$y = \overbrace{\begin{bmatrix}2 & 6\end{bmatrix}}^{\boldsymbol{C}}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \overbrace{0}^{\boldsymbol{D}}u$$

with the initial conditions

$$x_0 \in \{(0,0),(1,0),(0,-1),(2,2)\}$$

is described by

$$y(t) = \boldsymbol{C}e^{\boldsymbol{A}t}x(0) + \int_0^t \boldsymbol{C}e^{\boldsymbol{A}(t-\tau)}\boldsymbol{B}u(\tau)d\tau + \boldsymbol{D}u(t)$$



Observation: The eigenvalues of $A = \begin{cases}-0.2500 + 0.7599i\\ -0.2500 - 0.7599i\end{cases}$ are complex conjugates with negative real parts. The system is stable and $y(t)$ converges to 0, regardless of the initial condition x_0.

Task 2 – Pole placement

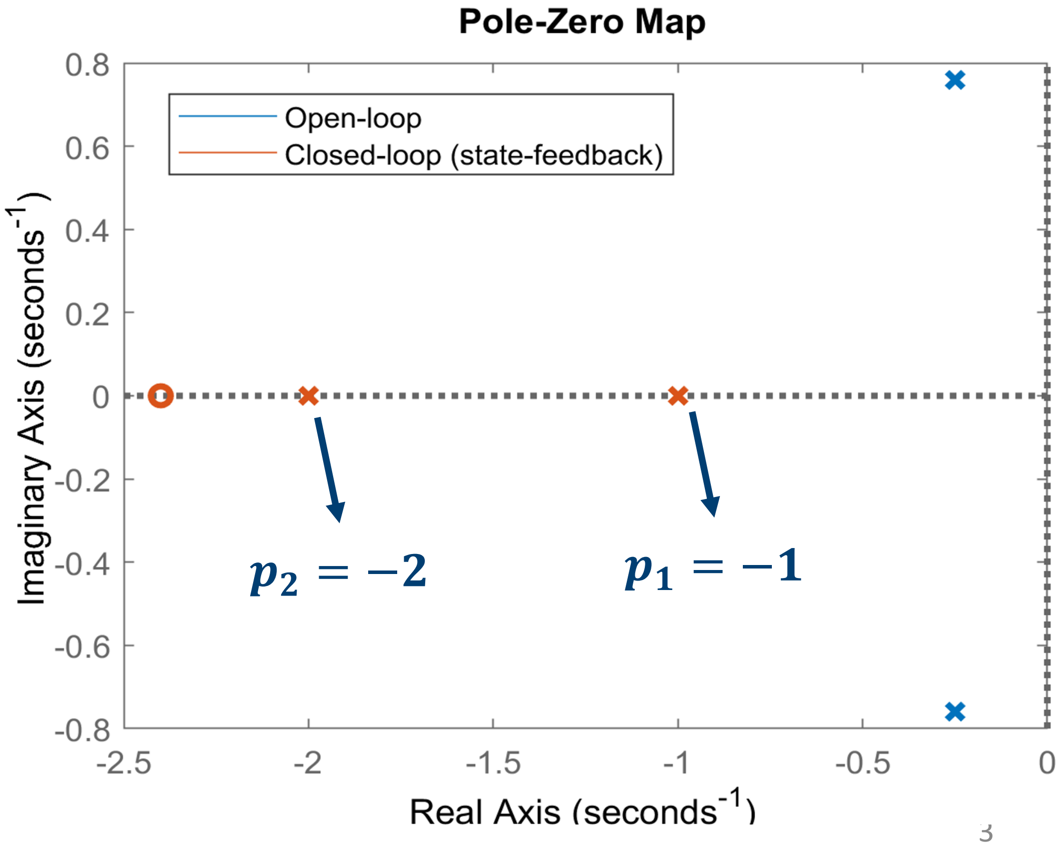
- **Pole placement** places the poles of a closed-loop system at the place of your choice to achieve the system characteristics you desire.
- Let's consider a state feedback $u(t) = [k_1 \quad k_2]x(t)$.
- Place the pole of the **closed-loop system** at -1 and -2:

```
K = place(A,B,[-1 -2]);
```

$$\begin{cases} \boldsymbol{k_1 = 2.5} \\ \boldsymbol{k_2 = 1.7} \end{cases} \qquad \begin{pmatrix} -0.5 & -0.8 \\ 0.8 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- The closed-loop system is described by

$$\dot{x} = Ax - Bu = (A - BK)x$$



Task 2 – closed-loop system dynamics

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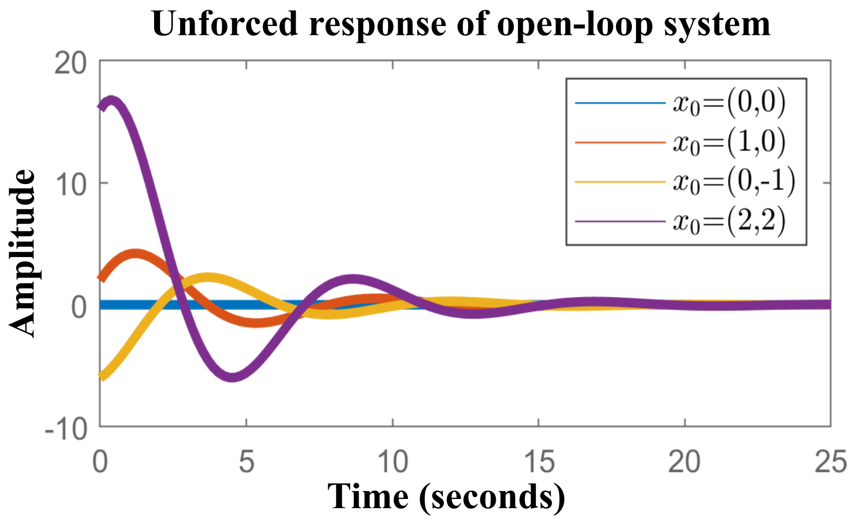
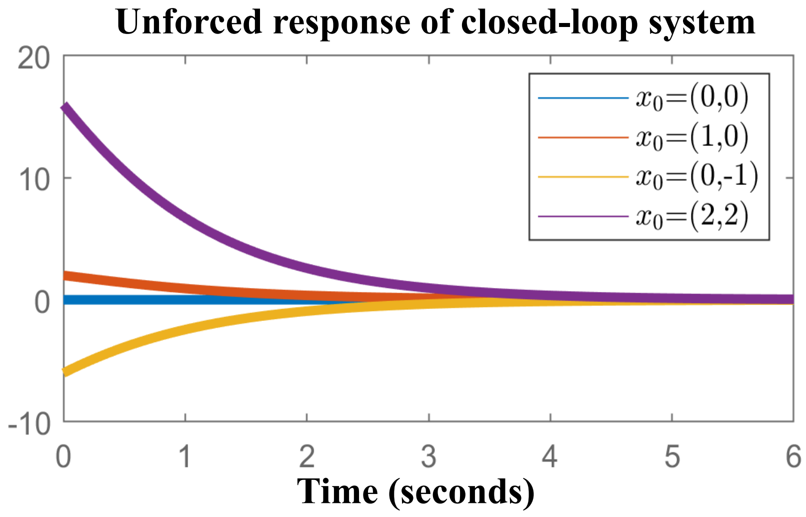
sys1 = ss(A, B, C, D)
```

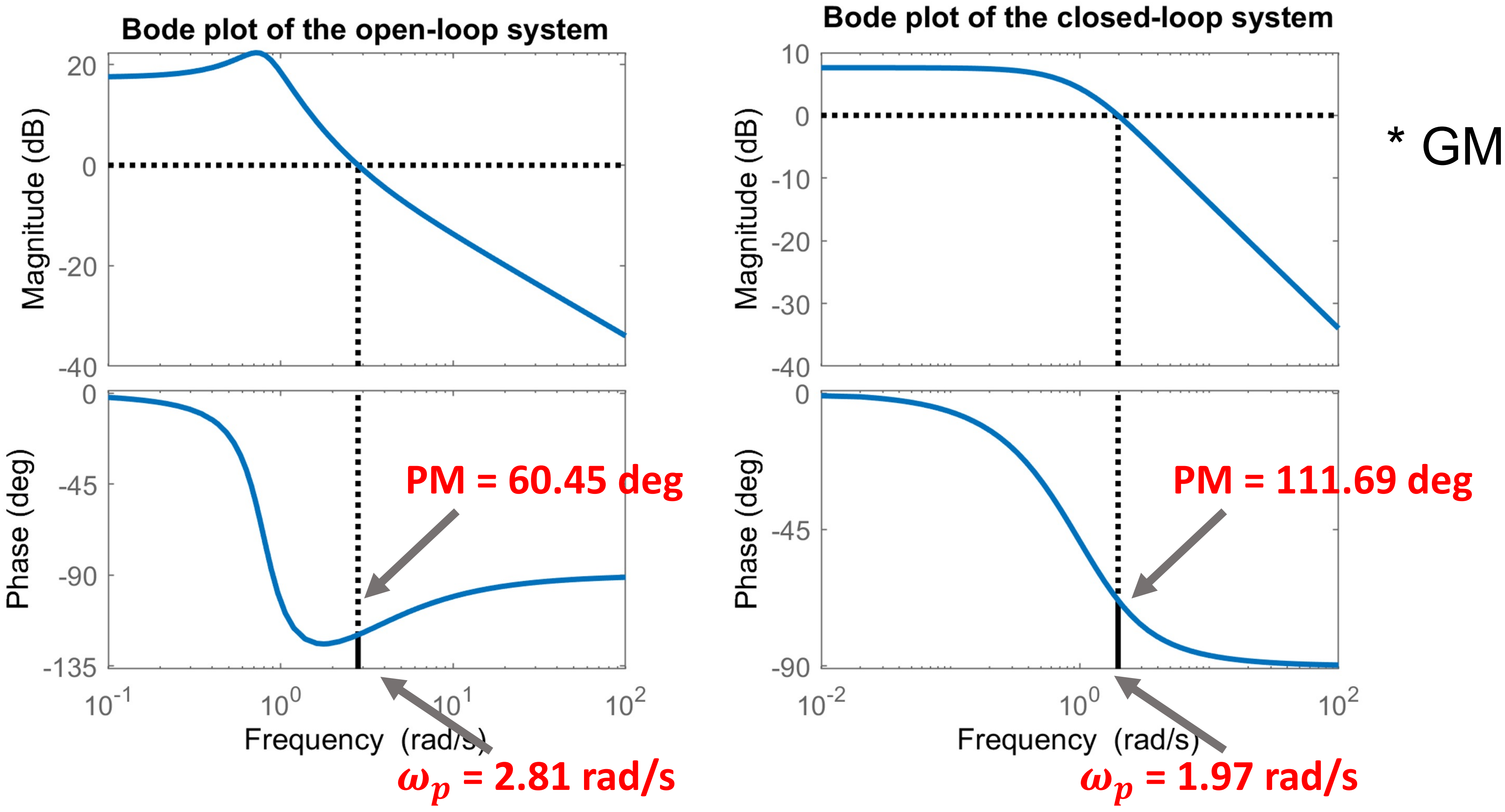



```
sys2 = ss(A-B*K, B, C, D)
```



- Both systems are **stable** and converge to 0
  - Poles of both systems have negative real parts.
- The closed-loop system (after pole placement) converges to the steady-state faster and has less overshoot
  - The closed-loop system exhibits our “desired” steady-state response.

Task 2 – Bode plots of the systems



* GM = Inf

The **closed-loop system** has a much larger phase margin!