

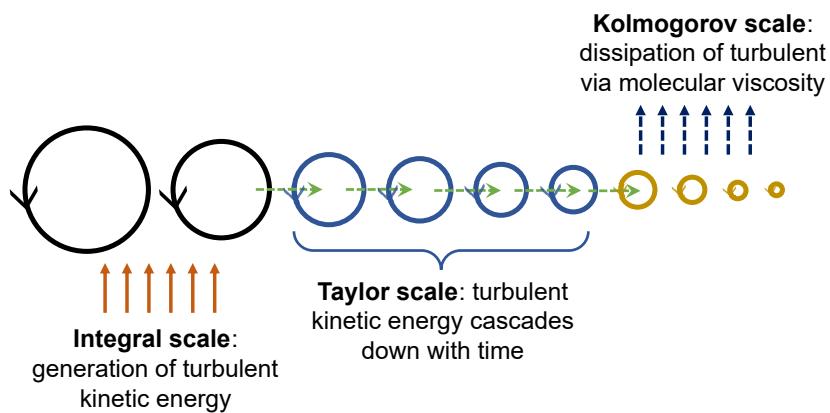
## 4.1 Turbulence

**The Reynolds number**  $Re$  measures the ratio of the inertial force to the viscous force. For **tube** flow,

$$Re = \frac{\rho U D}{\mu} = \frac{U D}{\nu} = \begin{cases} < 2000, & \text{laminar} \\ 2000 - 3000, & \text{transient} \\ > 3000, & \text{turbulence} \end{cases}.$$

### Turbulence characteristics

- **Random variation** of fluid properties (e.g., pressure and velocities) in time and space. Each property has a specific continuous energy spectrum which drops to zero at high wave numbers;
- **Eddies** or fluid packets of many sizes, which intermingle and fill the shear layers down to the smallest scale (as defined by Kolmogorov);
- **Self-sustaining motion** – once triggered, turbulent flow can maintain itself by producing new eddies to replace those lost to viscous dissipation;
- **Mixing** – rapid convection of mass, momentum and energy, much stronger than laminar flows.

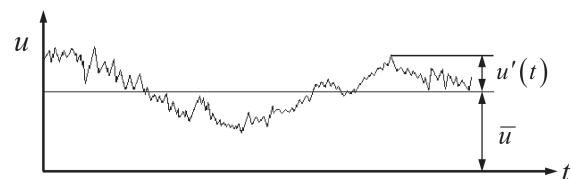


**FIG. 1:** Cascade of turbulence kinetic energy. The turbulence kinetic energy is generated on the integral scale (large eddies) and dissipated on the Kolmogorov scale (small eddies).

**Reynolds averaging** Instantaneous turbulent quantities can be measured, but turbulence is most usefully characterised statistically by decomposing flow quantities into mean and fluctuating components:

$$u(t) = \bar{u} + u'(t).$$

average velocity	velocity fluctuation
$\bar{u} = \frac{1}{T} \int_0^T u dt$	$u'(t) = u(t) - \bar{u}$



Hence, the  $x$ -momentum equation becomes (similar for  $y$ - and  $z$ -momentum equations),

$$\rho \frac{D\bar{u}}{Dt} = -\frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial x} \left( \mu \frac{\partial \bar{u}}{\partial x} - \rho \bar{u}' \bar{u}' \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial \bar{u}}{\partial y} - \rho \bar{u}' v' \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial \bar{u}}{\partial z} - \rho \bar{u}' w' \right) + \rho f_x,$$

where the  $\tau_{\text{turb}} = \rho \bar{u}' \bar{u}'$  terms are referred to as the **Reynolds stresses** (9 terms in total). The total shear stress  $\tau_{\text{total}} = \tau_{\text{lam}} + \tau_{\text{turb}}$ , where  $\tau_{\text{lam}} = \mu \frac{\partial \bar{u}}{\partial y}$ .

## 4.2 Energy Equation

**The Bernoulli equation** The Bernoulli equation assumes the fluid is incompressible and inviscid, the flow is steady, laminar, and no energy loss. It is applied between two points lying on the same streamline,

$$\frac{p_1}{\rho g} + \frac{1}{2g} u_1^2 + h_1 = \frac{p_2}{\rho g} + \frac{1}{2g} u_2^2 + h_2.$$

The Bernoulli equation can be interpreted as the conservation of mechanical energy in *frictionless* flow.

### The pipe flow energy equation

$$\frac{p_1}{\rho g} + \frac{1}{2g} u_1^2 + h_1 = \frac{p_2}{\rho g} + \frac{1}{2g} u_2^2 + h_2 + h_f,$$

where  $h_f = f \frac{L}{D} \frac{U^2}{2g}$ , denotes the **major head loss**, is added to the Bernoulli equation. This term is the energy loss due to fluid friction, where  $f$  is the Darcy friction factor. The presence of  $h_f$  leads to the pressure drop:  $\Delta p = h_f \rho g$ .

- For the flow in a circular pipe, if the flow is laminar (essentially, Poiseuille flow),  $f = 64/Re$ ;
- If the flow is turbulent, one needs to consult the Moody diagram, where the friction factor is related to the Reynolds number and the relative wall roughness of the pipe,  $f(Re, \frac{\epsilon}{d})$ .

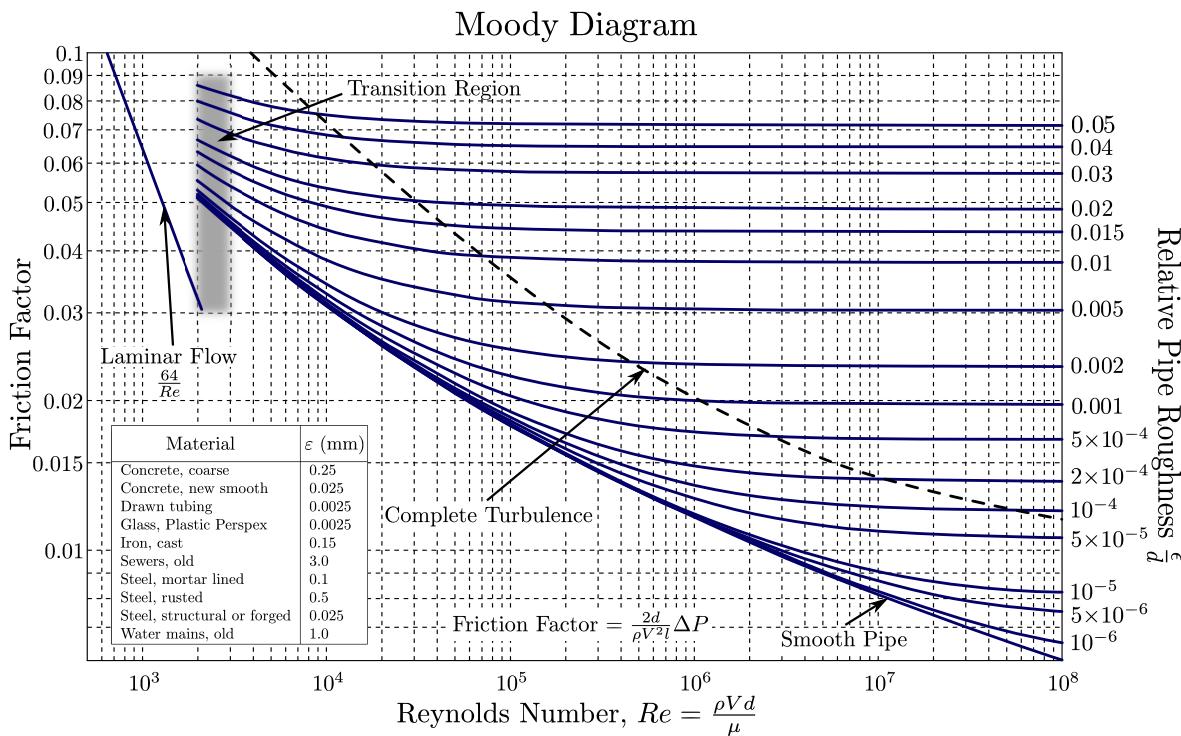


FIG. 2: Moody chart. (Wikipedia)

**Contraction and expansion loss** Energy losses are also associated with expansion/contraction in channel size and bends etc. This type of energy loss is known as the **minor head loss**<sup>1</sup>, which leads to the pressure drop

$$\Delta p = \rho g h_f \quad \Rightarrow \quad R = \frac{\Delta p}{Q} = \frac{\rho g h_f U}{A} = \frac{\rho g K_L U}{2g A}.$$

where the value of loss coefficient  $K_L$  can be found in Figure 3. Note that Figure 3 only applies for the turbulent flow in a pipe  $\Rightarrow$  always calculate  $Re$  before reading values from the chart!

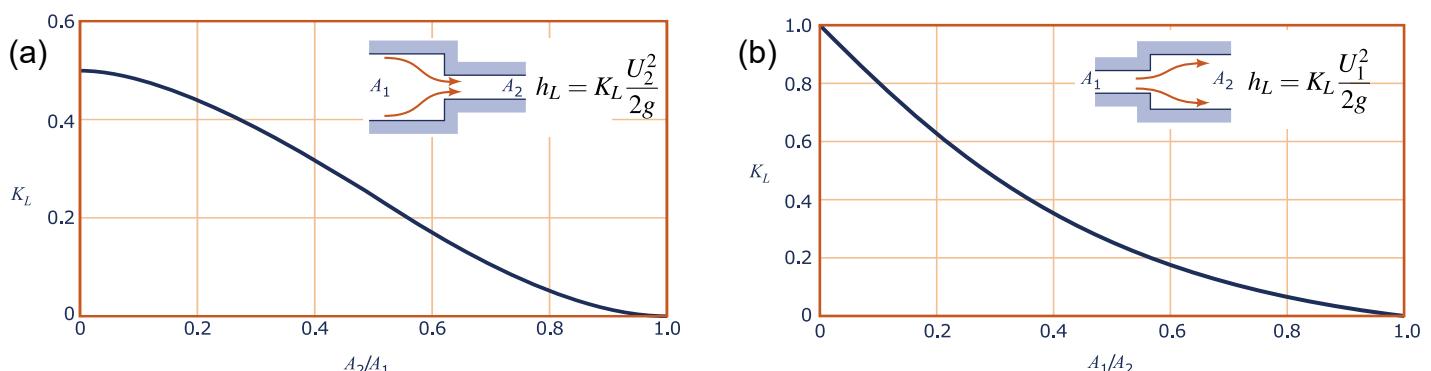


FIG. 3: Loss coefficient for a sudden (a) contraction, (b) expansion. (Munson et al.)