Prerequisite: Some Key Concepts of Fluid Mechanics

1. **Definition**: Fluid deforms indefinitely under an applied shear force, while solid resists the shear forces.

2. Commonly used fluid **properties**: velocity $\mathbf{u}\sim$ $[\mathrm{m/s}]$, pressure $p\sim$ $[\mathrm{Pa}] = [\mathrm{N/m^2}]$, density $\rho\sim$ $[\mathrm{kg/m^3}]$, energy $E\sim$ $[\mathrm{J/m^3}]$, dynamic viscosity $\mu\sim$ $[\mathrm{Pa\cdot s}] = [\mathrm{kg\cdot m^{-1}s^{-1}}]$, kinematic visvosity $\nu = \mu/\rho\sim$ $[\mathrm{m^2/s}]$. Note that in a higher dimension, velocity is a vector, but pressure is a scalar.

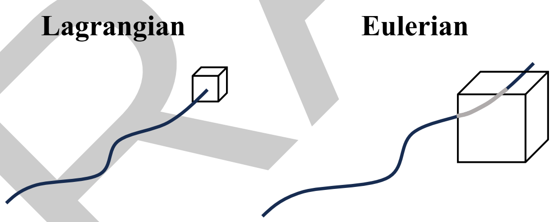
3. Two types of **stresses** that form the overall stress exerted on fluid boundaries:
   - **Hydrostatic** stress acts perpendicularly on the wall by the fluid at rest due to static pressure: $p = p_0 - \rho g z$.
   - **Deviatoric** stress acts tangentially on the wall that causes fluid flow or deformation due to the shear: $\tau = \mu \dfrac{\partial u}{\partial z}$ $[\mathrm{Pa}]$.

4. **Bernoulli equation**: to apply, flow must be assumed steady, incompressible (ρ is constant), inviscid ($\mu = 0$), no work done on the fluid, laminar; then:
$$p + \frac{1}{2}\rho u^2 + \rho g z = \text{constant} \tag{1}$$
where the second term quantifies the fluid kinetic energy per unit volume, and the third term quantifies the potential energy per unit volume.

5. **The Reynolds number**, $\mathrm{Re} = \dfrac{\rho u L}{\mu}$, is a dimensionless number that quantifies the relative importance of inertial and viscous effects. For an internal flow in a circular pipe, flow is generally believed to be turbulent when $\mathrm{Re} > 4000$, laminar when $\mathrm{Re} < 2300$, and transitional in between.

6. **Vorticity** of fluid flow: $\boldsymbol{\omega} = \nabla \times \mathbf{u}$, where $\nabla\times$ denotes the curl operation. If $\boldsymbol{\omega} = 0$, the flow is irrotational.

7. **Descriptions of motion**:
   - *Lagrangian*: keeps track of individual particles as they move through space; "go with the flow".
   - *Eulerian*: observe the rate of change of a property at fixed spatial locations.



8. **Reynolds Transport Theorem (RTT)**: a principle that describes the conservation of a physical quantity 'B' in a control volume (CV) deformed over time. Define $\beta = \mathrm{d}B/\mathrm{d}m$ as the amount of B per unit mass, RTT is
$$\frac{\mathrm{d}B_{\text{system}}}{\mathrm{d}t} = \frac{\partial}{\partial t}\int_{CV} \rho\beta \mathrm{d}V + \oint_{CS} \rho\beta(\mathbf{u}\cdot\hat{\mathbf{n}})\mathrm{d}A \tag{2}$$
In other words, $\left(\begin{array}{c}\text{Rate of change of}\\ B \text{ in the system}\end{array}\right) = \left(\begin{array}{c}\text{Rate of change of}\\ B \text{ in control volume}\end{array}\right) + \left(\begin{array}{c}\text{Net flux of } B \text{ out of}\\ \text{control volume}\end{array}\right)$.

9. The concept of RTT leads to the formulation of **Navier-Stokes** equations, which depicts the conservation of mass (B is mass) and momentum (B is linear momentum). Mathematically,
$$\text{(mass)} \qquad \nabla\cdot\mathbf{u} = 0 \tag{3}$$
$$\text{(momentum)} \qquad \rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u} + \rho\mathbf{f} \tag{4}$$
where $\nabla\cdot$ and ∇ denotes the divergence and gradient operation, respectively. These equations will be discussed extensively in the upcoming lectures; Let us save some words here. **Stay tuned!**