4.1 Turbulence

The Reynolds number Re measures the ratio of the momentum force to the viscus force. For **tube** flow,

$$\mathrm{Re} = \frac{\rho U D}{\mu} = \frac{U D}{\nu} = \begin{cases} < 2000, & \text{laminar} \\ 2000 - 3000, & \text{transient} \\ > 3000, & \text{turbulence} \end{cases}.$$

Turbulence characteristics

- **Random variation** of fluid properties (*e.g.*, pressure and velocities) in time and space. Each property has a specific continuous energy spectrum which drops to zero at high wave numbers;
- **Eddies** or fluid packets of many sizes, which intermingle and fill the shear layers down to the smallest scale (as defined by Kolmogorov);
- **Self-sustaining motion** – once triggered, turbulent flow can maintain itself by producing new eddies to replace those lost to viscous dissipation;
- **Mixing** – rapid convection of mass, momentum and energy, much stronger than laminar flows.

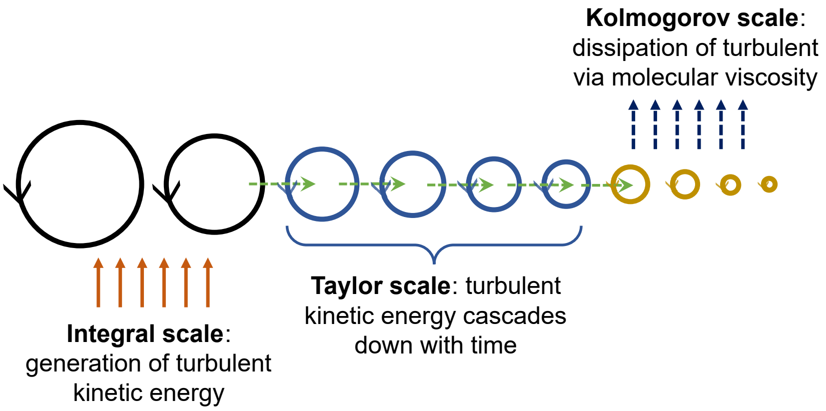
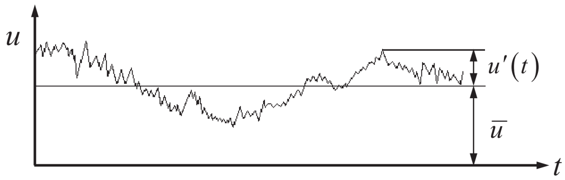


FIG. 1: Cascade of turbulence kinetic energy. The turbulence kinetic energy is generated on the integral scale (large eddies) and dissipated on the Kolmogorov scale (small eddies).

Reynolds averaging Turbulence cannot be measured. It can only be *characterised* in a *statistical* manner by decomposing a certain flow quantity (*e.g.*, velocity) into the mean and standard deviation (fluctuation) components.

$$u(t) = \bar{u} + u'(t).$$

average velocity	velocity fluctuation
$\bar{u} = \frac{1}{T}\int_0^T u\mathrm{d}t$	$u' = u - \bar{u}$



Hence, the x-momentum equation becomes (similar for y- and z-momentum equations),

$$\rho\frac{D\bar{u}}{Dt} = -\frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial x}\left(\mu\frac{\partial \bar{u}}{\partial x} - \rho\overline{u'u'}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial \bar{u}}{\partial y} - \rho\overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial \bar{u}}{\partial z} - \rho\overline{u'w'}\right) + \rho f_x,$$

where the $\tau'_{ij} = \rho\overline{u'_i u'_j}$ terms are referred to as the **Reynolds stresses** (9 terms in total), which need to be resolved with appropriate turbulence closure methodologies (*e.g.*, RANS k-ε model subjected to the Boussinesq approximation).

4.2 Energy Equation

The Bernoulli equation The Bernoulli equation assumes the fluid is incompressible and inviscid, the flow is steady, laminar, and no energy loss. It is applied between two points lying on the same streamline,

$$\frac{p_1}{\rho g} + \frac{1}{2g}u_1^2 + z_1 = \frac{p_2}{\rho g} + \frac{1}{2g}u_2^2 + z_2.$$

The Bernoulli equation can be interpreted as the conservation of mechanical energy in *frictionless* flow.

The pipe flow energy equation

$$\frac{p_1}{\rho g} + \frac{1}{2g}u_1^2 + z_1 = \frac{p_2}{\rho g} + \frac{1}{2g}u_2^2 + z_2 + h_f,$$

where $h_f = f\frac{L}{D}\frac{U^2}{2g}$, denotes the **major head loss**, is added to the Bernoulli equation. This term is the energy loss due to fluid friction, where f is the Darcy friction factor. The presence of h_f leads to the pressure drop: $\Delta p = h_f \rho g$.

- For the flow in a circular pipe, if the flow is laminar (essentially, Poiseuille flow), $f = 64/\mathrm{Re}$;
- If the flow is turbulent, one needs to consult the Moody diagram, where the friction factor is related to the Reynolds number and the relative wall roughness of the pipe, $f\left(\mathrm{Re}, \frac{\varepsilon}{d}\right)$.

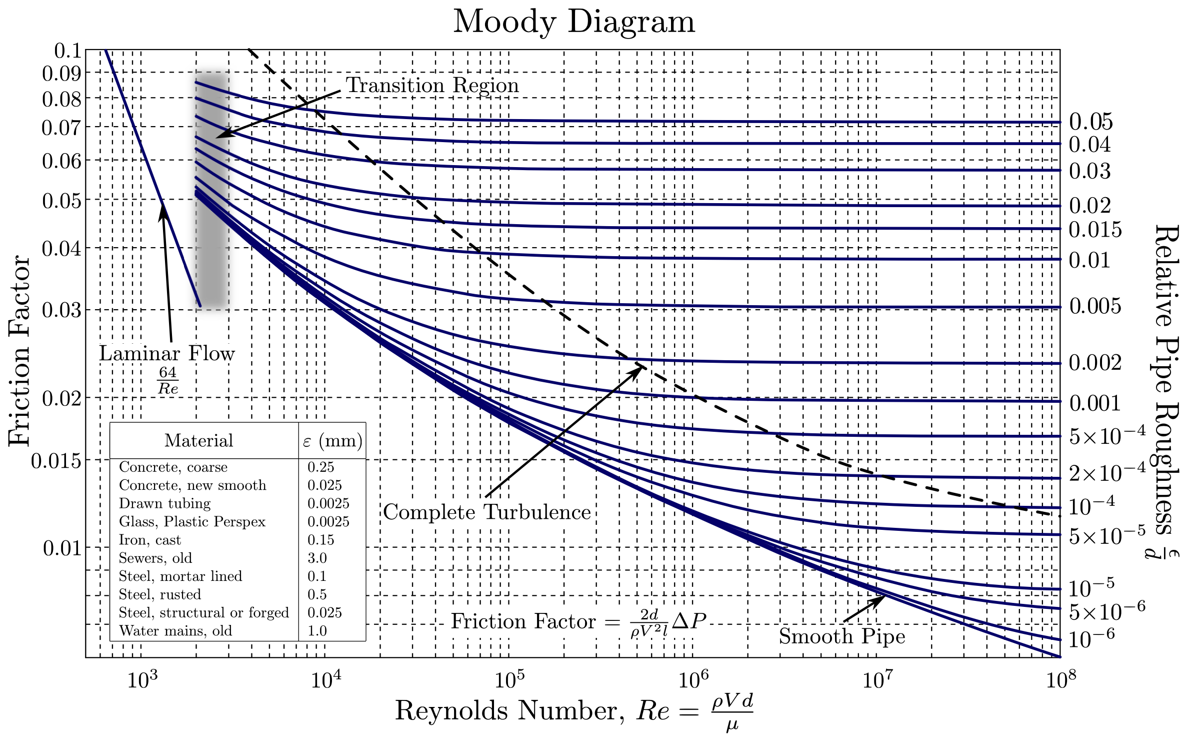


FIG. 2: Moody chart. (Wikipedia)

Contraction and expansion loss Energy losses are also associated with expansion/contraction in channel size and bends *etc.* This type of energy loss is known as the **minor head loss**[1], which leads to the pressure drop

$$\Delta p = \rho g h_f \quad \Rightarrow \quad R = \frac{\Delta p}{Q} = \frac{\rho g h_f U}{A} = \frac{\rho g K_L U}{2gA}.$$

where the value of loss coefficient K_L can be found in Figure 3. Note that Figure 3 only applies for the turbulent flow in a pipe ⇒ always calculate Re before reading values from the chart!

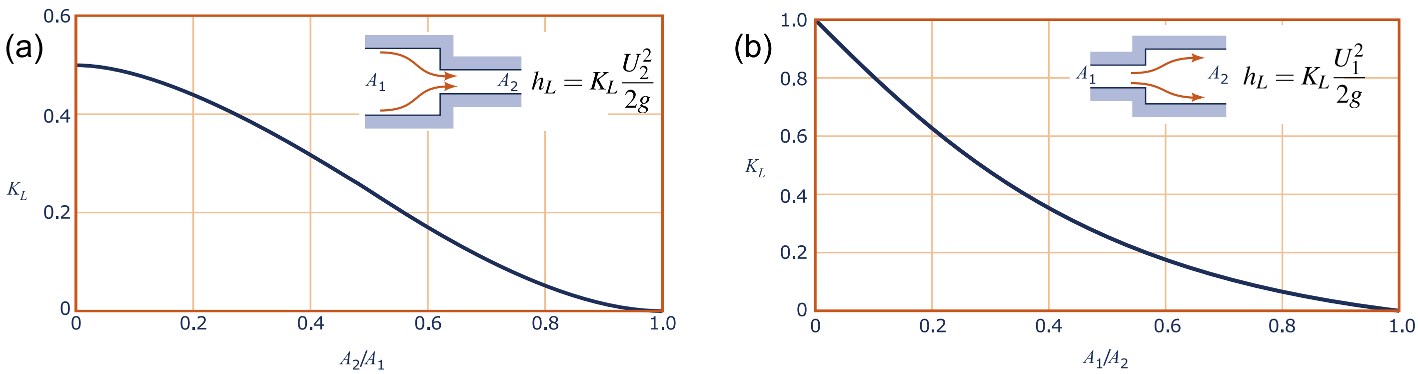


FIG. 3: Loss coefficient for a sudden (a) contraction, (b) expansion. (Munson *et al.*)

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[1]"major" and "minor" do not necessarily reflect the relative importance of each type of loss. The minor loss can be larger than the major loss.