1 Mean, Median and Standard Deviation

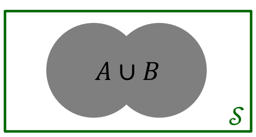
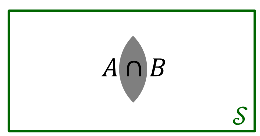
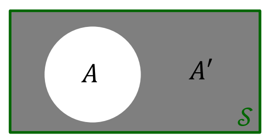
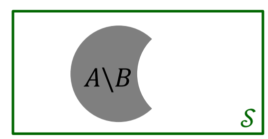
Mean	Median	Standard Deviation	MAD
$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + ... + x_n}{n}$	$\tilde{x} = \begin{cases} x(\frac{n+1}{2}), & n \text{ is odd} \\ 0.5(x(\frac{n}{2}) + x(\frac{n}{2}+1)), & n \text{ is even} \end{cases}$	$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$	$MAD = k\lvert\widetilde{x_i - \hat{x}}\rvert$ $k = 1.4826$ (normal)

- s, s^2 and $\bar{x}$ denote sample S.D., variance and mean.
- σ, σ^2 and μ denote population S.D., variance and mean [1].

- **Trimmed Mean at $p\%$** - Remove $p/2$ percentile from both ends of the data and calculate the mean
- **Standard Error on the mean** - $S_{\bar{X}} = \frac{S_X}{\sqrt{n}}$, where n is the sample size, S_X is the standard deviation

2 Probability Theory

Set Theory

Union	Intersection	Complement	Relative Complement
A OR B	A AND B	NOT A	A AND NOT B
			
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$		$P(A') = 1 - P(A)$	

Counting

- Product Rule - $n = n_1 \times n_2 \times n_3 \times ... \times n_k$
- Permutations: Ordered, dependent selections (without replacement) - $P_{k,n} = \frac{n!}{(n-k)!}$
- Combinations: Unordered selections (without replacement) - $C_{k,n} = \frac{P_{k,n}}{k!} = \binom{n}{k} = \frac{n!}{k!(n-k)!}$

Conditional Probability

- Conditional probability: knowing that A has happened, the probability of B given A - $P(B|A) = \frac{P(B \cap A)}{P(A)}$
- Bayes Theorem - $P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{k} P(B|A_i)P(A_i)}$
  (Applies if events A are mutually exclusive and exhaustive (defines the whole set)).

Independence

- Independent: event A is unaffected by event B

$$P(A \cap B) = P(A)P(B) \quad \Rightarrow \quad P(A|B) = P(A), \qquad P(B|A) = P(B)$$

[1] Mean is simply the average value of a *given sample*. Expect value is used when we want to calculate the mean of a *probability distribution*.

3 Probability Distributions and Discrete Random Variables

Binomial and Poisson Distribution

	Binomial Distribution	Poisson Distribution
Probability Density Function	$b(x;n,p) = \binom{n}{x} p^x (1-p)^{n-x}$	$po(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$
Expected Value, $E(x)$	$E(x) = np$	$E(x) = \lambda$
Variance, $V(x)$	$V(x) = np(1-p) = npq$	$V(x) = \lambda(1 - \frac{\lambda}{n}) \approx \lambda$

Note:

$V(x)$ can also be expressed as $V(x) = E(x^2) - (E(x))^2$.

Discrete Random Variables

Bernoulli RV	Expected Value	Variance
$p(x;\alpha) = \begin{cases} 1-\alpha, & if\ x=0 \\ \alpha, & if\ x=1 \\ 0, & otherwise \end{cases}$	$E(x) = \mu_X = \sum_{x \in D} xp(x)$	$V(X) = \sigma_X^2 = \sum_{x \in D} (x-\mu_X)^2 p(x) = E[(X-\mu_X)^2]$

4 Continuous Random Variables and Joint Probability

Random Variables

Probability density function	$P(a \le X \le b) = \int_a^b f(x)dx$
Cumulative distribution function	$F(x) = P(X \le x) = \int_{-\infty}^{x} f(y)dy$ and $\int_{-\infty}^{+\infty} f(x)dx = 1$
Expected value	$\mu_X = E(X) = \int_{-\infty}^{+\infty} xf(x)dx$
Variance	$\sigma_X^2 = V(X) = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx$

Joint Probability Distributions

- Discrete Joint random variables

Joint probability mass function (PMF)	Expected Value	Marginal PMFs
$p(x,y) = P(X = x$ and $Y = y)$ $\sum_x \sum_y p(x,y) = 1$	$E[h(X,Y)] = \sum_X \sum_Y h(x,y)p(x,y)$	$P_X(x) = \sum_y p(x,y)$ $P_Y(y) = \sum_x p(x,y)$

- Continuous Joint random variables

Joint probability mass function (PDF)	Expected Value	Marginal PDFs
$P[(X,Y) \in A] = \int\int_A f(x,y)dxdy$	$E[h(X,Y)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(x,y)f(x,y)dxdy$ $P[a \le X \le b, c \le Y \le d] = \int_c^d \int_a^b f(x,y)dxdy$	$f_X(x) = \int_{-\infty}^{+\infty} f(x,y)dy$ $f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)dx$

Independence

X and Y are independent if for every pair x, y:

- Discrete: $p(x,y) = p_x(x)p_y(y)$
- Continuous: $f(x,y) = f_x(x)f_y(y)$

5 Normal Distribution

- Probability Density Function (pdf):

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Cumulative Distribution Function (cdf):

$$F(X) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{x-\mu}{\sqrt{2\sigma^2}}\right)\right]$$

where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

Discrete	Continuous
$X = A$	$A - 0.5 < Y < A + 0.5$
$X \leq A$	$Y < A + 0.5$
$X < A$	$Y < A - 0.5$
$X \geq A$	$Y > A - 0.5$
$X > A$	$Y > A + 0.5$

Continuity Correction - Normal approximation of discrete distributions (e.g. Binomial)

Standard Normal Distribution

Standard normal distribution is a special case of normal distribution with $\mu = 0$ and $\sigma = 1$.

Probability Density Function (PDF)	Cumulative Distribution Function (CDF)
$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$	$\Phi(z) = F(Z) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{z}{\sqrt{2}}\right)\right]$ and $Z = \frac{X-\mu}{\sigma}$

Central Limit Theorem

For sufficiently large n:

- Mean sample mean is the population mean - $E[\bar{X}] = \mu_{\bar{X}} = \mu$
- Sample mean variance decreases as sample size increase - $V[\bar{X}] = \sigma^2_{\bar{X}} = \frac{\sigma^2}{n}$
- Standard error on the mean (SEM) is the SD of sample means - $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

Margin of Error, Confidence Intervals

Unknown σ, large n.

Confidence interval (CI)	Margin of Error (ME)
$CI = \bar{X}[\bar{X}_l, \bar{X}_u] \ = \ \left[\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \ \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right]$	$ME = \bar{X} \pm ME_{\bar{X}} = \lvert Z_{\frac{\alpha}{2}} \rvert \frac{\sigma}{\sqrt{n}}$

t-distribution

Unknown σ, small n. Test statistic - $T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$.

Confidence interval (CI)	Margin of Error (ME)
$CI = \bar{X}[\bar{X}_l, \bar{X}_u] \ = \ \left[\bar{X} - t_{\lvert \frac{\alpha}{2}, n-1 \rvert} \frac{s}{\sqrt{n}}, \ \bar{X} + t_{\lvert \frac{\alpha}{2}, n-1 \rvert} \frac{s}{\sqrt{n}}\right]$	$ME = \bar{X} \pm ME_{\bar{X}} = \lvert Z_{\frac{\alpha}{2}, n-1} \rvert \frac{s}{\sqrt{n}}$

χ^2-distribution and F-distribution

- χ^2-distribution is obtained by squaring normal distribution.
- For n observations with unknown μ and σ, test statistic - $\frac{(n-1)S^2}{\sigma^2}$
- F-statistic is the ratio of two variances (see ANOVA).

$$F(\nu_1, \nu_2) = \chi^2_1(\nu_1)/\chi^2_2(\nu_2)$$

6 Hypothesis Testing

NHST Types of Errors

$\mathbf{H_0}$	True	False
Do not reject	True negative, $1-\alpha$	False negative, β
Reject	False positive, α	True positive, $1-\beta$

- α denotes the probability of type I error.
- β denotes the probability of type II error.

z-test & t-test

- Assumptions:
  - Data are sampled from a normal distribution. Often lognormal distribution is more appropriate (Use SW or AD test to confirm)
- Null Hypothesis H_0 (for 2-tailed tests):
  - one sample: no differences between the mean and a population mean.
  - two sample: no differences between the two sample mean.
  - p-value is an estimate of the probability of getting a test-statistic more extreme. Reject H_0 if $p \leq \alpha$.

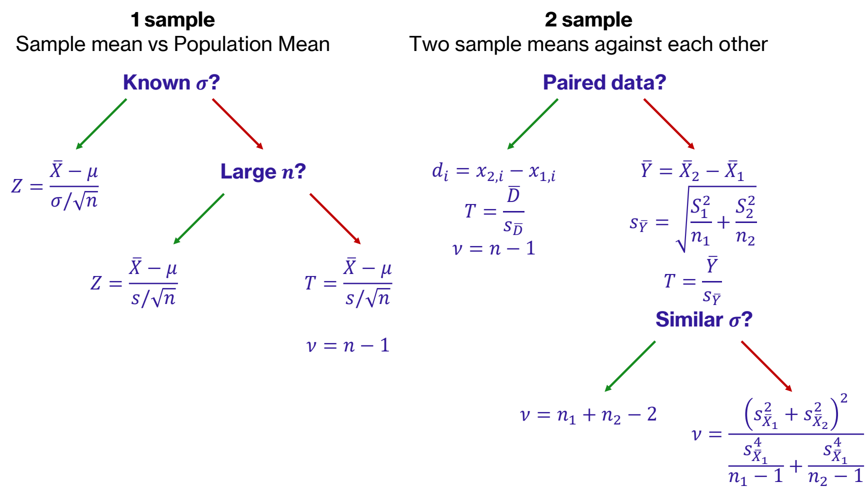
	Expression	Notes
z-test statistic	$Z = \dfrac{X-\mu}{\sigma/\sqrt{n}}$	$\dfrac{\text{effect}}{\text{error}}$, same below
t-test one-sample statistic	$T = \dfrac{\bar{X}_B - \mu_A}{s_B/\sqrt{n}}$	$v = n-1$
t-test two-sample statistic	$T = \dfrac{\bar{X}_B - \bar{X}_A}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}}$	assume similar variances, $v = n_A + n_B - 2$
Welch-Satterthwaite Equation	$\nu = \dfrac{(s_{\bar{X}_1}^2 + s_{\bar{X}_2}^2)^2}{\frac{s_{\bar{X}_1}^4}{n_1-1} + \frac{s_{\bar{X}_2}^4}{n_2-1}}$	$s_{\bar{X}_1}^2 = \frac{s_1^2}{n_1}, \quad s_{\bar{X}_2}^2 = \frac{s_2^2}{n_2}$

- Sample size estimation: for z-tests only
  - One tailed:

$$\beta(\mu_1) = \Phi\left(z_\alpha + \frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}}\right) \qquad n = \left[\frac{\sigma(z_\alpha + z_\beta)}{\mu_0 - \mu_1}\right]^2$$

  - Two tailed:

$$\beta(\mu_1) = \Phi\left(z_{\alpha/2} + \frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}}\right) - \Phi\left(-z_{\alpha/2} + \frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}}\right) \qquad n = \left[\frac{\sigma(z_{\alpha/2} + z_\beta)}{\mu_0 - \mu_1}\right]^2$$



ANOVA

Assumptions:

- Data are sampled from a normal distribution. Often lognormal distribution is more appropriate (Use SW or AD test to confirm)
- All groups have similar variance (homoscedastic) (Use Levene's or Bartlett's test to confirm)
- Hypothesis H_0: no differences between the means of any of the groups

One-way ANOVA

- Sample means and grand mean:

Sample Means	Grand Mean
$\bar{X}_i = \frac{1}{J}\sum_{j=1}^{J} X_{i,j}$	$\bar{X} = \frac{1}{N}\sum_{i=1}^{I}\sum_{j=1}^{J} X_{i,j} = \frac{1}{I}\sum_{i=1}^{I}\bar{X}_i$

  - I denotes the number of groups
  - J_i denotes the number of observations for a given group.
  - N is the total number of observations, $N = \sum_{i=1}^{I} J_i$.
  - $X_{i,j}$ is a random variable that denotes the j^{th} observation from the i^{th} group.

- Sums of Squares: $SS_T = SS_R + SS_M$

	Sum of Squares	DoF	Notes
Total	$SS_T = \sum_{i=1}^{I}\sum_{j=1}^{J_i}(X_{i,j} - \bar{X})^2 = (N-1)S^2_{grand}$	$\nu_T = N - 1$	$S^2_{grand} = \frac{1}{N-1}\sum_{i=1}^{I}\sum_{j=1}^{J_i}(X_{i,j} - \bar{X})^2$
Model	$SS_M = \sum_{i=1}^{I} J_i(\bar{X}_i - \bar{X})^2$	$\nu_M = I - 1$	
Residuals	$SS_R = \sum_{i=1}^{I}\sum_{j=1}^{J_i}(X_{i,j} - \bar{X}_i)^2 = \sum_{i=1}^{I} S_i^2(J_i - 1)$	$\nu_R = N - I$	$S_i^2 = \frac{\sum_{j=1}^{J_i}(X_{i,j} - \bar{X}_i)^2}{J_i - 1}$

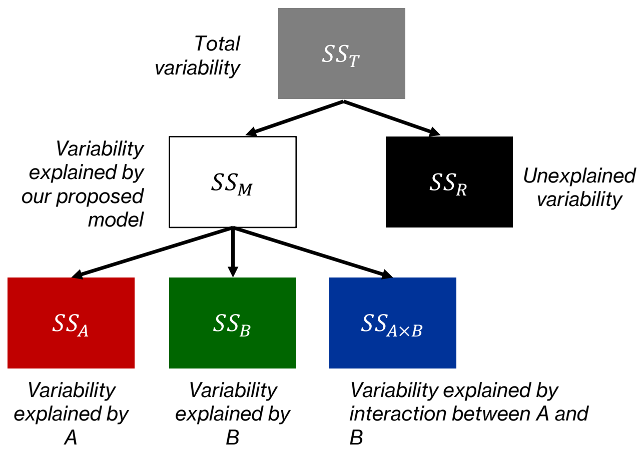
- Mean square values and test-statistic:

$$MS_M = \frac{SS_M}{\nu_M} = \frac{1}{I-1}\sum_{i=1}^{I} J_i(\bar{X}_i - \bar{X})^2 \qquad MS_R = \frac{SS_R}{\nu_R} = \frac{1}{N-I}\sum_{i=1}^{I}\sum_{j=1}^{J_i}(X_{i,j} - \bar{X}_i)^2$$

$$F = \frac{MS_M}{MS_R} \qquad p = 1 - f(F, \nu_M, \nu_R)$$

Two-way ANOVA

	Sum of Squares	DoF	Mean Square	F-Statistic	Notes
Total	$SS_T = \sum_{g=1}^{G}\sum_{i=1}^{I}\sum_{j=1}^{J}(X_{g,i,j} - \bar{X})^2$	$\nu_T = N - 1$			$N = GIJ$
Model	$SS_M = \sum_{g=1}^{G}\sum_{i=1}^{I} J(\bar{X}_{gi} - \bar{X})^2$	$\nu_M = GI - 1$			
Residuals	$SS_R = \sum_{g=1}^{G}\sum_{i=1}^{I}\sum_{j=1}^{J}(X_{g,i,j} - \bar{X}_{gi})^2$	$\nu_R = N - GI$	$MS_R = \frac{SS_R}{\nu_R}$		
SS_A	$SS_A = IJ\sum_{g=1}^{G}(\bar{X}_g - \bar{X})^2$	$\nu_A = G - 1$	$MS_A = \frac{SS_A}{\nu_A}$	$F_A = \frac{MS_A}{MS_R}$	SS_M grouped by G
SS_B	$SS_B = GJ\sum_{i=1}^{I}(\bar{X}_i - \bar{X})^2$	$\nu_B = I - 1$	$MS_B = \frac{SS_B}{\nu_B}$	$F_B = \frac{MS_B}{MS_R}$	SS_M grouped by I
$SS_{A\times B}$	$SS_{A\times B} = SS_M - SS_A - SS_B$	$\nu_{A\times B} = \nu_M - \nu_A - \nu_B$	$MS_{A\times B} = \frac{SS_{A\times B}}{\nu_{A\times B}}$	$F_{A\times B} = \frac{MS_{A\times B}}{MS_R}$	left over SS



Post-Hoc Tests

- Bonferroni - pairwise t-tests. Statistically significant if $p < p_{crit} = \frac{\alpha}{I}$.
- Tukey Kramer Test (less conservative than Bonferroni).

Non-parametric Tests

- **Sign test**: one sample, asymmetric data.
  - Compare each sample to a pre-defined median value $\tilde{\mu}_0$: +ve/-ve difference?
  - Make hypothesis:
    * H_0: the population median $\tilde{\mu}$ from which the sample is taken is equal to $\tilde{\mu}_0$.
    * H_1: $\tilde{\mu} \neq \tilde{\mu}_0$.
  - If H_0 is true, we should obtain at least $n/2$ of data points that are negative.
  - find P by taking binomial test by taking $p = 0.5$.

- **Wilcoxon Sign-rank test**: one sample, non-symmetric data.
  - Compare each sample to a pre-defined median value $\tilde{\mu}_0$. Consider both the **sign** and **magnitude** of differences.
  - Sort and find the rank of the differences. Multiply the ranks by their sign.

rank	...	...	...
|difference|	...	...	...
sign	...	...	...
signed rank	...	...	...

  - calculate the absolute sum of the signed ranks, for +ve and -ve, respectively.
    * $W^+ = \sum_i^{n^+} R_i^+$ and $W^- = \sum_i^{n^-} -R_i^-$
    * the smaller one becomes the test statistic.
  - Critical value W_{crit} can be located from the table.

- **Mann-Whitney U-test**: two samples.
  - Make hypothesis:
    * H_0: the mean rank of the two levels are equal. (If samples are independent are from similar underlying distributions : mean rank -> median.)
    * H_1: the mean rank of the two levels are different.

- Group both samples, rank and correct averages **in one table**.

rank	...	...	...
data value	...	...	...

- Calculate the sum ranks for each group W.

$$W_X = \sum_{i}^{n_X} R_i^X \qquad W_Y = \sum_{i}^{n_Y} R_i^Y$$

- Calculate U-statistic for each group

$$U_i = W_i - \frac{n_i(n_i+1)}{2}$$

- For $n < 8$: U_{crit} can be found from the table. Reject H_0 if $min(U_1, U_2) < u_{crit}$
- For $n \geq 8$:

$$\mu_U = \frac{n_1 n_2}{2} \qquad \sigma_U^2 = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}$$

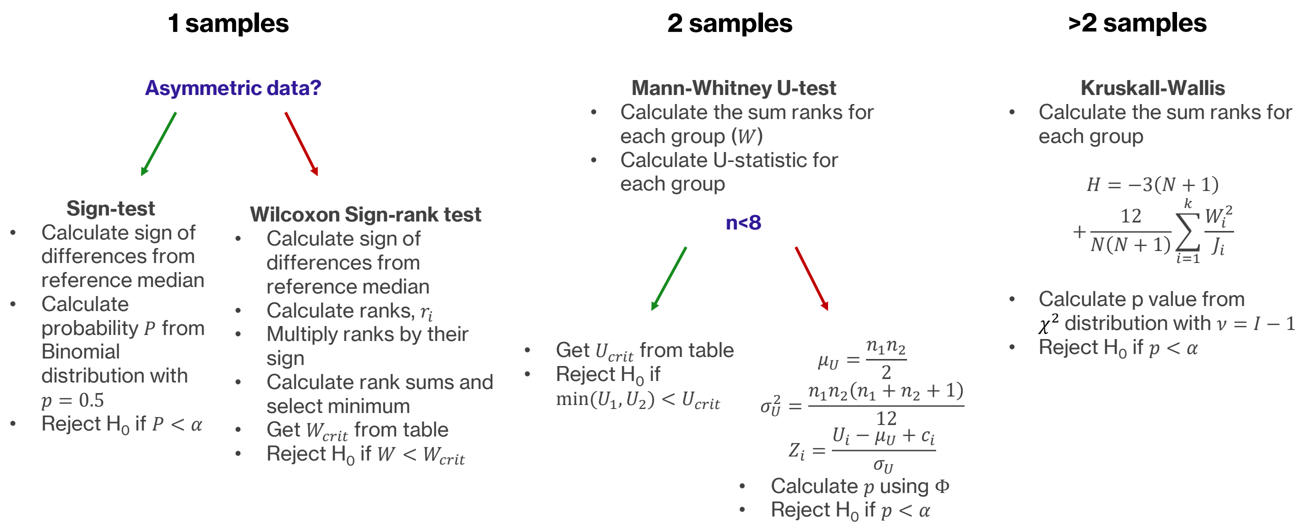
z-statistic can be calculated by

$$Z_i = \frac{U_i - \mu_U(+continuity)}{\sigma_U}$$

- **Kruskall-Wallis test**: more than two samples.
  - H_0: the I independent random samples all come from identical populations.
  - Calculate the sum ranks for each group:

$$H = -3(N+1) + \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{W_i^2}{J_i}$$

  - For all $J_i > 5$, and if H_0 is true, H is approximated to χ^2 distribution with $\nu = I - 1$.



7 Correlation and Regression

Correlation

Covariance (discrete and continuous)	Population Correlation Coefficient
$cov(X,Y) = E[(x-\mu_X)(y-\mu_Y)] = E(XY) - E(X)E(Y) = \begin{cases} \sum_x \sum_y (x-\mu_X)(y-\mu_Y)p(x,y) \\ \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (x-\mu_X)(y-\mu_Y)f(x,y)dxdy \end{cases}$	$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$

- **Pearson's correlation coefficient**: r is an point estimator of $R = \hat{\rho}$. Assume linear relationship between X and Y.

$$\sigma_X^2 = S_{XX} = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}, \qquad \sigma_Y^2 = S_{YY} = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}, \qquad cov(x,y) = S_{XY} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})^2}{n-1}$$

$$\Rightarrow \quad r_{X,Y} = \frac{S_{XY}}{\sqrt{S_{XX}}\sqrt{S_{YY}}} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}} \quad \in [-1, 1]$$

- **Test statistic and DoF**

$$T = \frac{R\sqrt{n-2}}{\sqrt{1-R^2}}, \qquad \nu = n-2$$

- **Spearman's rank correlation coefficient**: non-linear relationship between X and Y - $\rho = 1 - \frac{6\sum(x_i - y_i)^2}{n(n^2-1)}$, where x_i, y_i are ranks. If x_i, y_i are tied, we evaluate Pearson's r on the ranks.

Regression

- **Regression model** - $Y = b_0 + b_1 X + \epsilon$, where ϵ is a normally distributed random variable with $E(\epsilon) = 0$ and $V(\epsilon) = \sigma_\epsilon^2$.
- **Estimate of the best fit line**

$\mathbf{Y} = \hat{\mathbf{b}}_\mathbf{0} + \hat{\mathbf{b}}_\mathbf{1}\mathbf{X}$

slope	$\hat{b}_1 = \frac{S_{xy}}{S_{xx}}$	$s_{\hat{b}_1} = \frac{s_\epsilon}{\sqrt{S_{xx}}}$
intercept	$\hat{b}_0 = \bar{y} - \hat{b}_1\bar{x}$	$s_{\hat{b}_0} = \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}}$

$\xrightarrow{\text{with}}$

S_{xx}	S_{yy}	S_{xy}
$\sum_{i=1}^n (x_i - \bar{x})^2$	$\sum_{i=1}^n (y_i - \bar{y})^2$	$\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$

  - Best estimator for variance of residuals - $\hat{\sigma}_\epsilon^2 = s_\epsilon^2 = \frac{SS_R}{\nu_R} = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2}$

- **Coefficient of determination**:

$$r^2 = 1 - \frac{SS_R}{SS_T}, \qquad \text{where} \quad SS_T = S_{yy}, \quad SS_R = s_\epsilon^2 \nu_R,$$

- **Confidence interval**:

slope	intercept	variance of residual
$CI = \hat{b}_1 + t_{1-\frac{\alpha}{2},\nu} s_{\hat{b}_1}$	$CI = \hat{b}_0 + t_{1-\frac{\alpha}{2},\nu} s_{\hat{b}_0}$	$s_\epsilon^2 = SS_R/\nu_R$ and $\nu_R = n-2$

- **Confidence bands**: For any given value of x^*, uncertainty on the mean

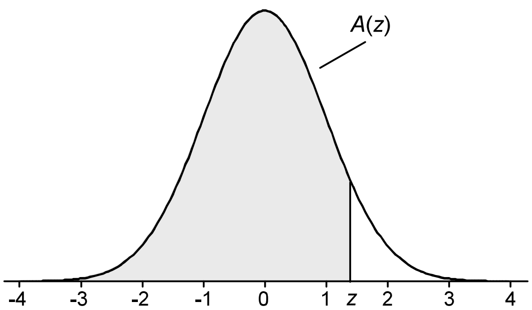
$$CI = \hat{y}^* \pm t_{\frac{\alpha}{2},n-1} s_{\hat{y}^*} \qquad s_{\hat{y}^*} = s_\epsilon \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2}}$$

- **Prediction bands**: uncertainty on a single measurement

$$s_{pred}^2 = s_{\hat{y}^*}^2 + s_\epsilon^2 \qquad PI = \hat{y}^* \pm t_{\frac{\alpha}{2},n-2} s_{pred}^*$$

Table A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

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TABLE A.2

t* Distribution: Critical Values of *t

Degrees of freedom	Two-tailed test: / One-tailed test:	10% / 5%	5% / 2.5%	2% / 1%	1% / 0.5%	0.2% / 0.1%	0.1% / 0.05%
	Significance level						
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291

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Table A.3

F Distribution: Critical Values of *F* (5% significance level)

ν_1 (ν_M) / ν_2 (ν_R)	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.36	246.46	247.32	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.71	8.69	8.67	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.87	5.84	5.82	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.64	4.60	4.58	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.96	3.92	3.90	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.53	3.49	3.47	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.24	3.20	3.17	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.03	2.99	2.96	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.86	2.83	2.80	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.74	2.70	2.67	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.64	2.60	2.57	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.55	2.51	2.48	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.48	2.44	2.41	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.42	2.38	2.35	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.37	2.33	2.30	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.33	2.29	2.26	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.29	2.25	2.22	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.26	2.21	2.18	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.22	2.18	2.15	2.12
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.20	2.16	2.12	2.10
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.17	2.13	2.10	2.07
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.15	2.11	2.08	2.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.13	2.09	2.05	2.03
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.11	2.07	2.04	2.01
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.09	2.05	2.02	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.08	2.04	2.00	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.06	2.02	1.99	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.05	2.01	1.97	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.04	1.99	1.96	1.93
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.99	1.94	1.91	1.88
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.95	1.90	1.87	1.84
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.89	1.85	1.81	1.78
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.86	1.82	1.78	1.75
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.84	1.79	1.75	1.72
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.82	1.77	1.73	1.70
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.80	1.76	1.72	1.69
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.79	1.75	1.71	1.68
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.78	1.73	1.69	1.66
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.76	1.71	1.67	1.64
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.80	1.74	1.69	1.66	1.62
250	3.88	3.03	2.64	2.41	2.25	2.13	2.05	1.98	1.92	1.87	1.79	1.73	1.68	1.65	1.61
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86	1.78	1.72	1.68	1.64	1.61
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85	1.78	1.72	1.67	1.63	1.60
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.77	1.71	1.66	1.62	1.59
600	3.86	3.01	2.62	2.39	2.23	2.11	2.02	1.95	1.90	1.85	1.77	1.71	1.66	1.62	1.59
750	3.85	3.01	2.62	2.38	2.23	2.11	2.02	1.95	1.89	1.84	1.77	1.70	1.66	1.62	1.58
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.76	1.70	1.65	1.61	1.58

TABLE A.3 (continued)

***F* Distribution: Critical Values of *F* (5% significance level)**

v_2 (v_R) \ v_1 (v_M)	25	30	35	40	50	60	75	100	150	200
1	249.26	250.10	250.69	251.14	251.77	252.20	252.62	253.04	253.46	253.68
2	19.46	19.46	19.47	19.47	19.48	19.48	19.48	19.49	19.49	19.49
3	8.63	8.62	8.60	8.59	8.58	8.57	8.56	8.55	8.54	8.54
4	5.77	5.75	5.73	5.72	5.70	5.69	5.68	5.66	5.65	5.65
5	4.52	4.50	4.48	4.46	4.44	4.43	4.42	4.41	4.39	4.39
6	3.83	3.81	3.79	3.77	3.75	3.74	3.73	3.71	3.70	3.69
7	3.40	3.38	3.36	3.34	3.32	3.30	3.29	3.27	3.26	3.25
8	3.11	3.08	3.06	3.04	3.02	3.01	2.99	2.97	2.96	2.95
9	2.89	2.86	2.84	2.83	2.80	2.79	2.77	2.76	2.74	2.73
10	2.73	2.70	2.68	2.66	2.64	2.62	2.60	2.59	2.57	2.56
11	2.60	2.57	2.55	2.53	2.51	2.49	2.47	2.46	2.44	2.43
12	2.50	2.47	2.44	2.43	2.40	2.38	2.37	2.35	2.33	2.32
13	2.41	2.38	2.36	2.34	2.31	2.30	2.28	2.26	2.24	2.23
14	2.34	2.31	2.28	2.27	2.24	2.22	2.21	2.19	2.17	2.16
15	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.10	2.10
16	2.23	2.19	2.17	2.15	2.12	2.11	2.09	2.07	2.05	2.04
17	2.18	2.15	2.12	2.10	2.08	2.06	2.04	2.02	2.00	1.99
18	2.14	2.11	2.08	2.06	2.04	2.02	2.00	1.98	1.96	1.95
19	2.11	2.07	2.05	2.03	2.00	1.98	1.96	1.94	1.92	1.91
20	2.07	2.04	2.01	1.99	1.97	1.95	1.93	1.91	1.89	1.88
21	2.05	2.01	1.98	1.96	1.94	1.92	1.90	1.88	1.86	1.84
22	2.02	1.98	1.96	1.94	1.91	1.89	1.87	1.85	1.83	1.82
23	2.00	1.96	1.93	1.91	1.88	1.86	1.84	1.82	1.80	1.79
24	1.97	1.94	1.91	1.89	1.86	1.84	1.82	1.80	1.78	1.77
25	1.96	1.92	1.89	1.87	1.84	1.82	1.80	1.78	1.76	1.75
26	1.94	1.90	1.87	1.85	1.82	1.80	1.78	1.76	1.74	1.73
27	1.92	1.88	1.86	1.84	1.81	1.79	1.76	1.74	1.72	1.71
28	1.91	1.87	1.84	1.82	1.79	1.77	1.75	1.73	1.70	1.69
29	1.89	1.85	1.83	1.81	1.77	1.75	1.73	1.71	1.69	1.67
30	1.88	1.84	1.81	1.79	1.76	1.74	1.72	1.70	1.67	1.66
35	1.82	1.79	1.76	1.74	1.70	1.68	1.66	1.63	1.61	1.60
40	1.78	1.74	1.72	1.69	1.66	1.64	1.61	1.59	1.56	1.55
50	1.73	1.69	1.66	1.63	1.60	1.58	1.55	1.52	1.50	1.48
60	1.69	1.65	1.62	1.59	1.56	1.53	1.51	1.48	1.45	1.44
70	1.66	1.62	1.59	1.57	1.53	1.50	1.48	1.45	1.42	1.40
80	1.64	1.60	1.57	1.54	1.51	1.48	1.45	1.43	1.39	1.38
90	1.63	1.59	1.55	1.53	1.49	1.46	1.44	1.41	1.38	1.36
100	1.62	1.57	1.54	1.52	1.48	1.45	1.42	1.39	1.36	1.34
120	1.60	1.55	1.52	1.50	1.46	1.43	1.40	1.37	1.33	1.32
150	1.58	1.54	1.50	1.48	1.44	1.41	1.38	1.34	1.31	1.29
200	1.56	1.52	1.48	1.46	1.41	1.39	1.35	1.32	1.28	1.26
250	1.55	1.50	1.47	1.44	1.40	1.37	1.34	1.31	1.27	1.25
300	1.54	1.50	1.46	1.43	1.39	1.36	1.33	1.30	1.26	1.23
400	1.53	1.49	1.45	1.42	1.38	1.35	1.32	1.28	1.24	1.22
500	1.53	1.48	1.45	1.42	1.38	1.35	1.31	1.28	1.23	1.21
600	1.52	1.48	1.44	1.41	1.37	1.34	1.31	1.27	1.23	1.20
750	1.52	1.47	1.44	1.41	1.37	1.34	1.30	1.26	1.22	1.20
1000	1.52	1.47	1.43	1.41	1.36	1.33	1.30	1.26	1.22	1.19

TABLE A.3 (continued)

***F* Distribution: Critical Values of *F* (1% significance level)**

ν_1 (ν_M) / ν_2 (ν_R)	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	6106.32	6142.67	6170.10	6191.53	6208.73
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.44	99.44	99.45
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.92	26.83	26.75	26.69
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.25	14.15	14.08	14.02
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.77	9.68	9.61	9.55
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.60	7.52	7.45	7.40
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.36	6.28	6.21	6.16
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.56	5.48	5.41	5.36
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	5.01	4.92	4.86	4.81
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.60	4.52	4.46	4.41
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.29	4.21	4.15	4.10
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.05	3.97	3.91	3.86
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.86	3.78	3.72	3.66
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.70	3.62	3.56	3.51
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.56	3.49	3.42	3.37
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.45	3.37	3.31	3.26
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.35	3.27	3.21	3.16
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.27	3.19	3.13	3.08
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.19	3.12	3.05	3.00
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.13	3.05	2.99	2.94
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.07	2.99	2.93	2.88
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	3.02	2.94	2.88	2.83
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.97	2.89	2.83	2.78
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.93	2.85	2.79	2.74
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.89	2.81	2.75	2.70
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.86	2.78	2.72	2.66
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.82	2.75	2.68	2.63
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.79	2.72	2.65	2.60
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.77	2.69	2.63	2.57
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.74	2.66	2.60	2.55
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.74	2.64	2.56	2.50	2.44
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.56	2.48	2.42	2.37
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.56	2.46	2.38	2.32	2.27
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.39	2.31	2.25	2.20
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.45	2.35	2.27	2.20	2.15
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.42	2.31	2.23	2.17	2.12
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52	2.39	2.29	2.21	2.14	2.09
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.37	2.27	2.19	2.12	2.07
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.23	2.15	2.09	2.03
150	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53	2.44	2.31	2.20	2.12	2.06	2.00
200	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.27	2.17	2.09	2.03	1.97
250	6.74	4.69	3.86	3.40	3.09	2.87	2.71	2.58	2.48	2.39	2.26	2.15	2.07	2.01	1.95
300	6.72	4.68	3.85	3.38	3.08	2.86	2.70	2.57	2.47	2.38	2.24	2.14	2.06	1.99	1.94
400	6.70	4.66	3.83	3.37	3.06	2.85	2.68	2.56	2.45	2.37	2.23	2.13	2.05	1.98	1.92
500	6.69	4.65	3.82	3.36	3.05	2.84	2.68	2.55	2.44	2.36	2.22	2.12	2.04	1.97	1.92
600	6.68	4.64	3.81	3.35	3.05	2.83	2.67	2.54	2.44	2.35	2.21	2.11	2.03	1.96	1.91
750	6.67	4.63	3.81	3.34	3.04	2.83	2.66	2.53	2.43	2.34	2.21	2.11	2.02	1.96	1.90
1000	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.20	2.10	2.02	1.95	1.90

TABLE A.3 (continued)

***F* Distribution: Critical Values of *F* (1% significance level)**

v_1 (v_M) / v_2 (v_R)	25	30	35	40	50	60	75	100	150	200
1	6239.83	6260.65	6275.57	6286.78	6302.52	6313.03	6323.56	6334.11	6344.68	6349.97
2	99.46	99.47	99.47	99.47	99.48	99.48	99.49	99.49	99.49	99.49
3	26.58	26.50	26.45	26.41	26.35	26.32	26.28	26.24	26.20	26.18
4	13.91	13.84	13.79	13.75	13.69	13.65	13.61	13.58	13.54	13.52
5	9.45	9.38	9.33	9.29	9.24	9.20	9.17	9.13	9.09	9.08
6	7.30	7.23	7.18	7.14	7.09	7.06	7.02	6.99	6.95	6.93
7	6.06	5.99	5.94	5.91	5.86	5.82	5.79	5.75	5.72	5.70
8	5.26	5.20	5.15	5.12	5.07	5.03	5.00	4.96	4.93	4.91
9	4.71	4.65	4.60	4.57	4.52	4.48	4.45	4.41	4.38	4.36
10	4.31	4.25	4.20	4.17	4.12	4.08	4.05	4.01	3.98	3.96
11	4.01	3.94	3.89	3.86	3.81	3.78	3.74	3.71	3.67	3.66
12	3.76	3.70	3.65	3.62	3.57	3.54	3.50	3.47	3.43	3.41
13	3.57	3.51	3.46	3.43	3.38	3.34	3.31	3.27	3.24	3.22
14	3.41	3.35	3.30	3.27	3.22	3.18	3.15	3.11	3.08	3.06
15	3.28	3.21	3.17	3.13	3.08	3.05	3.01	2.98	2.94	2.92
16	3.16	3.10	3.05	3.02	2.97	2.93	2.90	2.86	2.83	2.81
17	3.07	3.00	2.96	2.92	2.87	2.83	2.80	2.76	2.73	2.71
18	2.98	2.92	2.87	2.84	2.78	2.75	2.71	2.68	2.64	2.62
19	2.91	2.84	2.80	2.76	2.71	2.67	2.64	2.60	2.57	2.55
20	2.84	2.78	2.73	2.69	2.64	2.61	2.57	2.54	2.50	2.48
21	2.79	2.72	2.67	2.64	2.58	2.55	2.51	2.48	2.44	2.42
22	2.73	2.67	2.62	2.58	2.53	2.50	2.46	2.42	2.38	2.36
23	2.69	2.62	2.57	2.54	2.48	2.45	2.41	2.37	2.34	2.32
24	2.64	2.58	2.53	2.49	2.44	2.40	2.37	2.33	2.29	2.27
25	2.60	2.54	2.49	2.45	2.40	2.36	2.33	2.29	2.25	2.23
26	2.57	2.50	2.45	2.42	2.36	2.33	2.29	2.25	2.21	2.19
27	2.54	2.47	2.42	2.38	2.33	2.29	2.26	2.22	2.18	2.16
28	2.51	2.44	2.39	2.35	2.30	2.26	2.23	2.19	2.15	2.13
29	2.48	2.41	2.36	2.33	2.27	2.23	2.20	2.16	2.12	2.10
30	2.45	2.39	2.34	2.30	2.25	2.21	2.17	2.13	2.09	2.07
35	2.35	2.28	2.23	2.19	2.14	2.10	2.06	2.02	1.98	1.96
40	2.27	2.20	2.15	2.11	2.06	2.02	1.98	1.94	1.90	1.87
50	2.17	2.10	2.05	2.01	1.95	1.91	1.87	1.82	1.78	1.76
60	2.10	2.03	1.98	1.94	1.88	1.84	1.79	1.75	1.70	1.68
70	2.05	1.98	1.93	1.89	1.83	1.78	1.74	1.70	1.65	1.62
80	2.01	1.94	1.89	1.85	1.79	1.75	1.70	1.65	1.61	1.58
90	1.99	1.92	1.86	1.82	1.76	1.72	1.67	1.62	1.57	1.55
100	1.97	1.89	1.84	1.80	1.74	1.69	1.65	1.60	1.55	1.52
120	1.93	1.86	1.81	1.76	1.70	1.66	1.61	1.56	1.51	1.48
150	1.90	1.83	1.77	1.73	1.66	1.62	1.57	1.52	1.46	1.43
200	1.87	1.79	1.74	1.69	1.63	1.58	1.53	1.48	1.42	1.39
250	1.85	1.77	1.72	1.67	1.61	1.56	1.51	1.46	1.40	1.36
300	1.84	1.76	1.70	1.66	1.59	1.55	1.50	1.44	1.38	1.35
400	1.82	1.75	1.69	1.64	1.58	1.53	1.48	1.42	1.36	1.32
500	1.81	1.74	1.68	1.63	1.57	1.52	1.47	1.41	1.34	1.31
600	1.80	1.73	1.67	1.63	1.56	1.51	1.46	1.40	1.34	1.30
750	1.80	1.72	1.66	1.62	1.55	1.50	1.45	1.39	1.33	1.29
1000	1.79	1.72	1.66	1.61	1.54	1.50	1.44	1.38	1.32	1.28