Imperial College London

BIOE50011 – Signals and ***Control***

MATLAB practical 2 – Poles and Stability

12 December, 2022

Learning objective

By the end of this MATLAB session you should be able to:

- Understand and derive the characteristics of electric circuit components: resistor *R*, capacitor *C*, inductor *L*
- Solve linear dynamic equations using the Laplace transform
- Determine how the system's dynamics depend on the poles of the transfer function
- Explain how the system components affect the poles in the *s*-plane

Poles and Stability – Recap

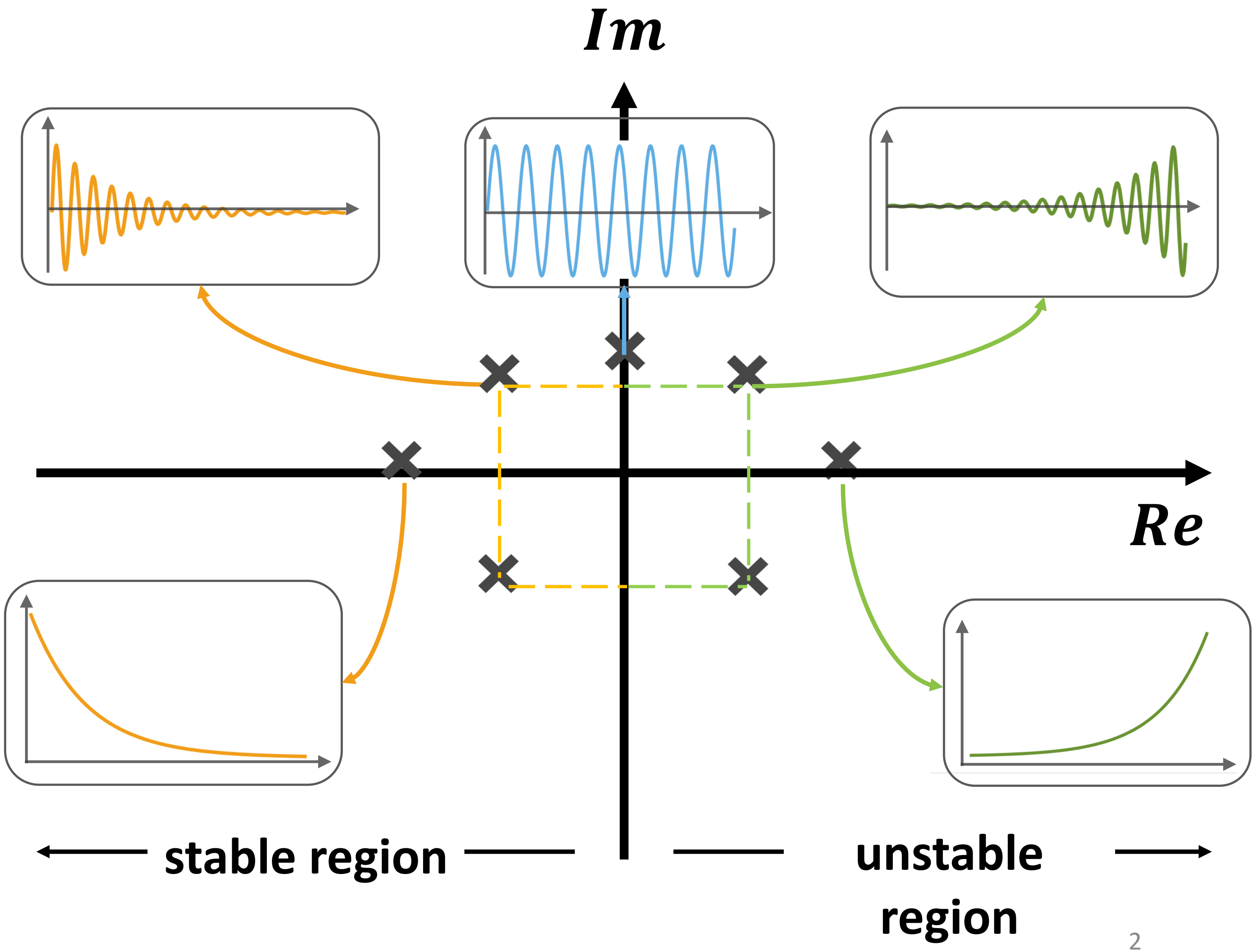
For a LTI system

$$G(s) = \frac{N(s)}{D(s)}$$

Poles, p_i are the solution of $D(s) = 0$

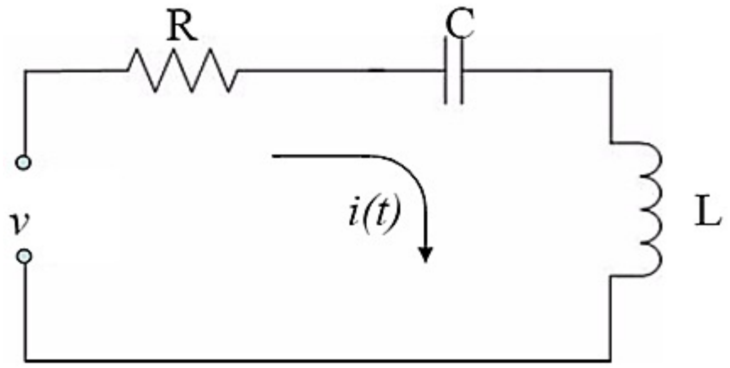
Three types of stability

- **Asymptotic stability**: $Re(p_i) < 0$
- **Instability**: $Re(p_i) > 0$
- **Marginal stability**: $Re(p_i) = 0$



Task 2 - RLC circuit

Different values of *R, L* and *C* result in different poles in the *s*-plane and influence the systems dynamics

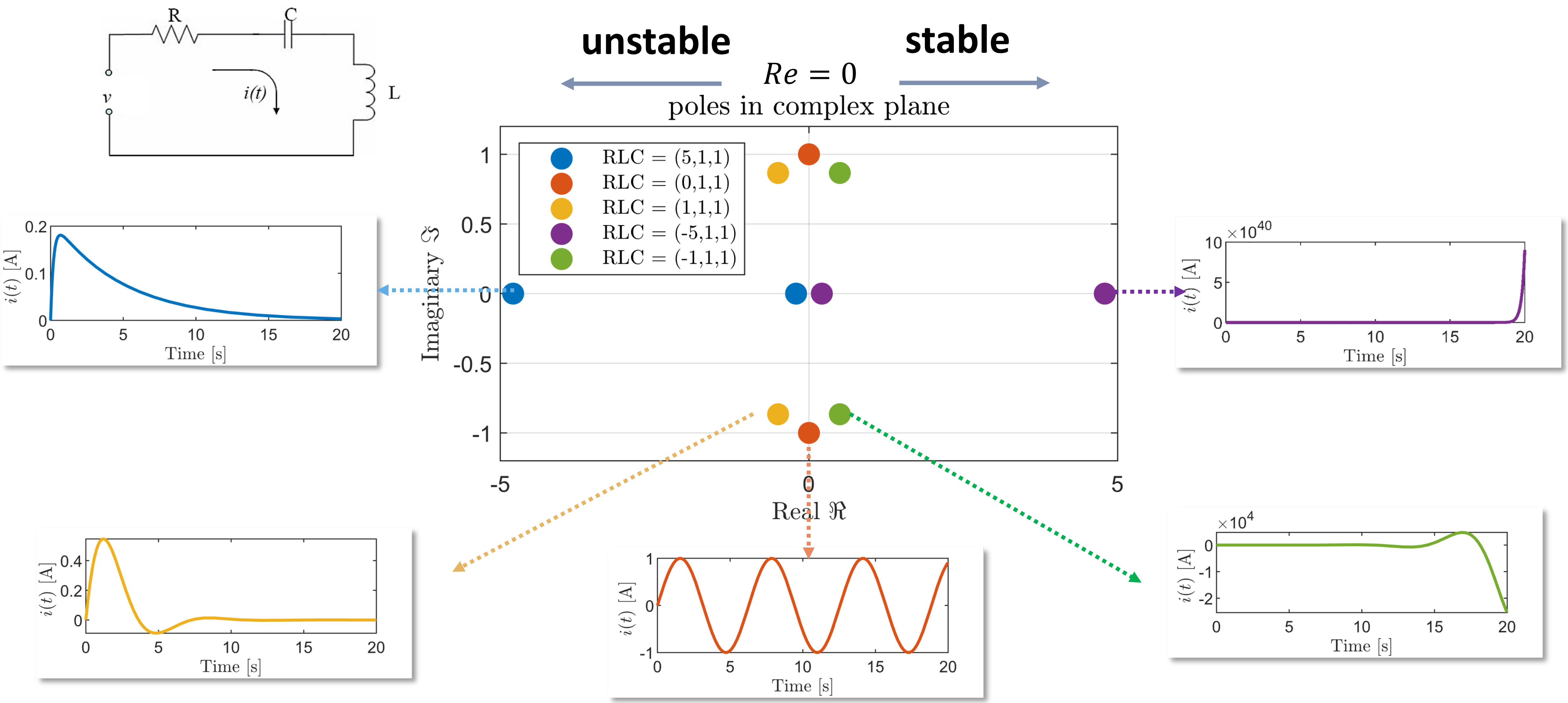


$$G(s) = \frac{I(s)}{V(s)} = \frac{\boldsymbol{Cs}}{\boldsymbol{LCs^2 + RCs + 1}}$$

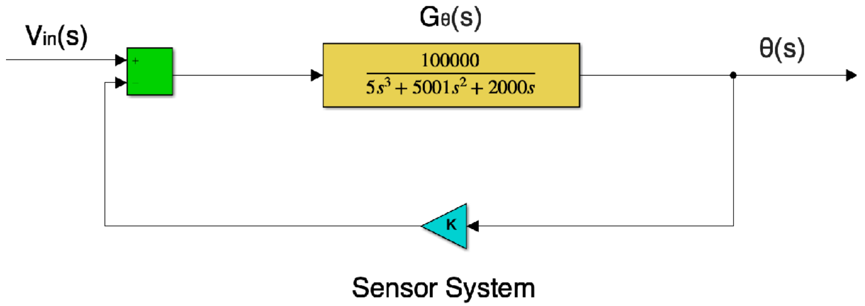
```
[Res, Pole] = residue([C],[C*L C*R 1]);

Current = Res(1) * exp(Pole(1)*t)
        + Res(2) * exp(Pole(2)*t);
```

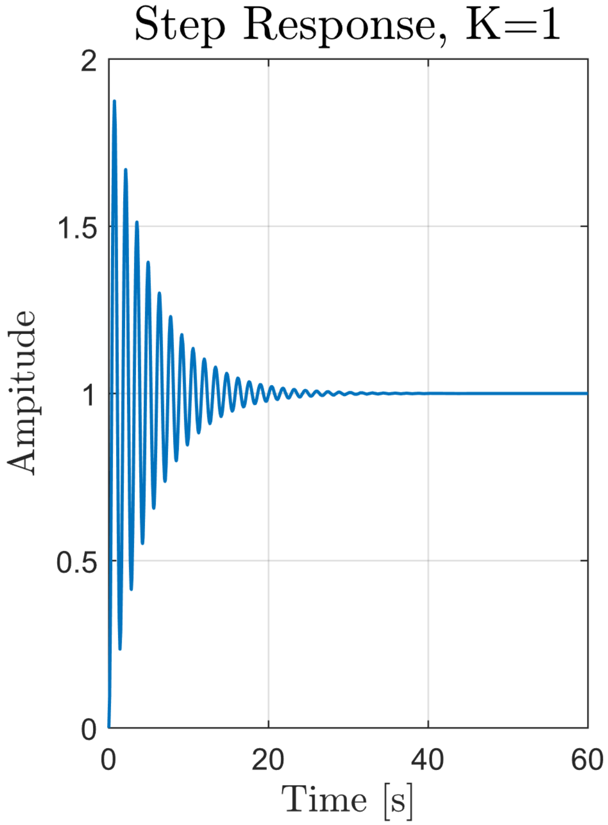
$$\boldsymbol{i(t) = a_1 e^{p_1 t} + a_2 e^{p_2 t}}$$

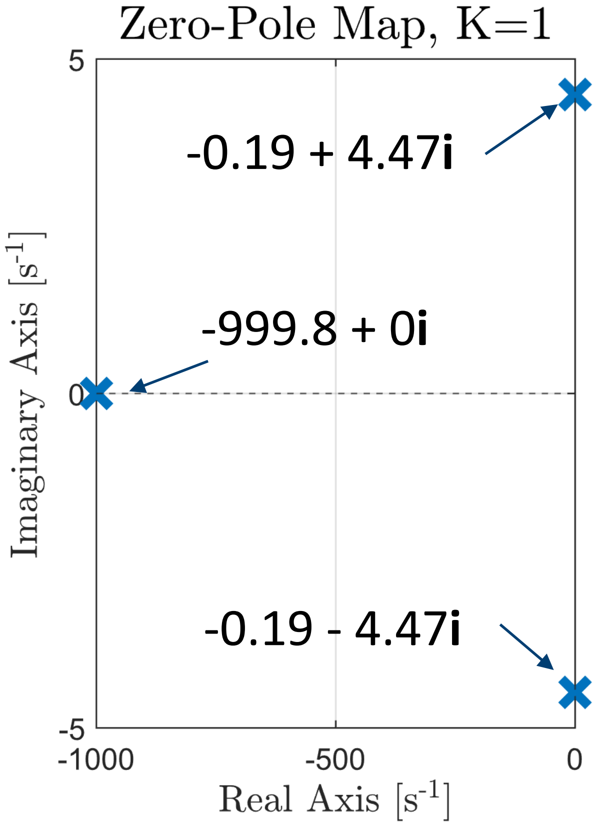
Task 2-Step responses and poles in the *s*-plane



Task 3 - Closed-loop control of the wheel system







$$G(s) = \frac{10^5}{5s^3 + 5001s^2 + 2000s + 10^5 K}$$

final value theorem

$$\lim_{s \to 0} s \frac{1}{s} G(s) = \frac{1}{K}$$

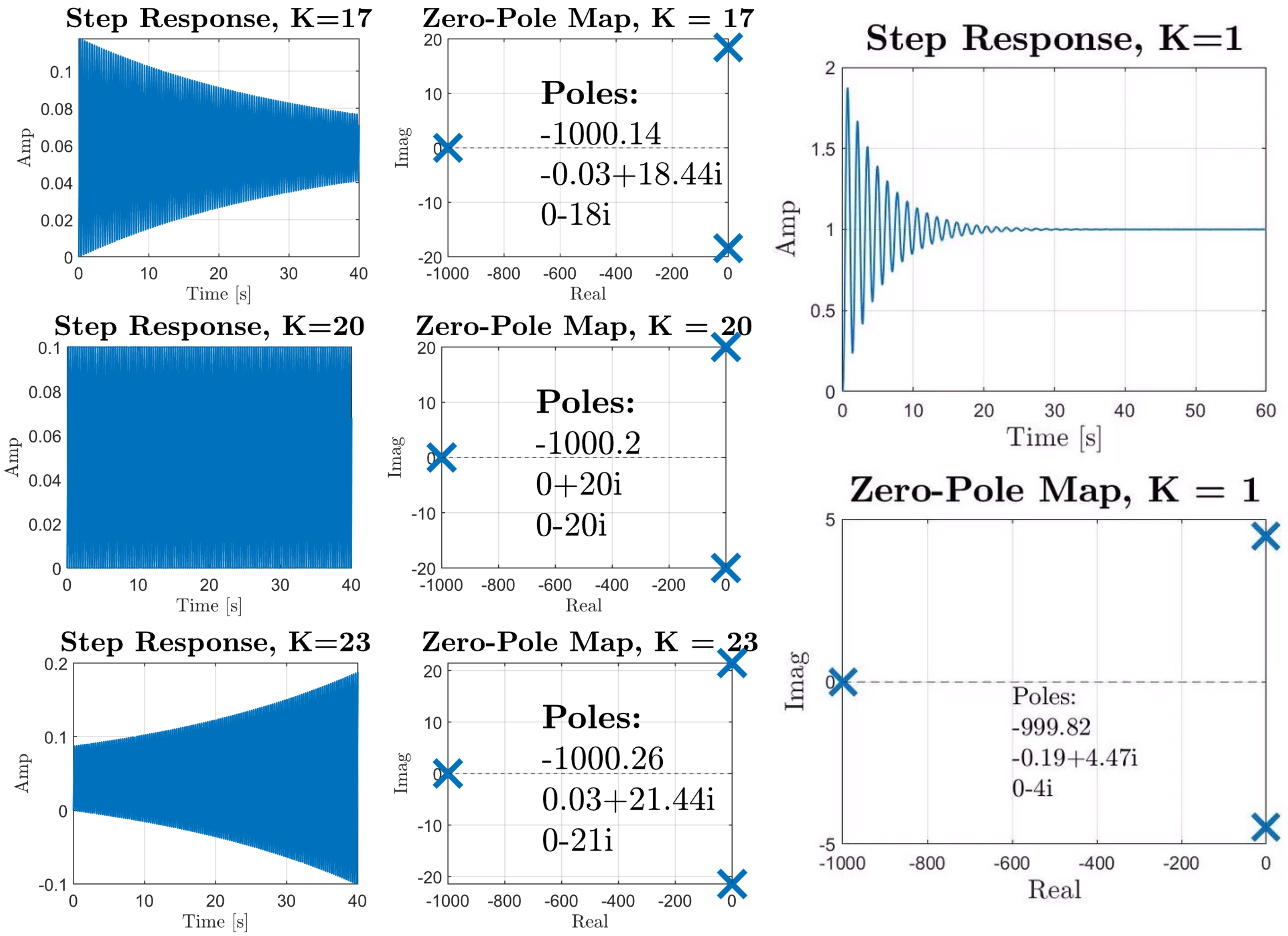
When K = 1

- all poles have a <u>negative real</u> part
- the step response converges to $\frac{1}{K} = 1$

Task 3 - Step response and poles for varied K

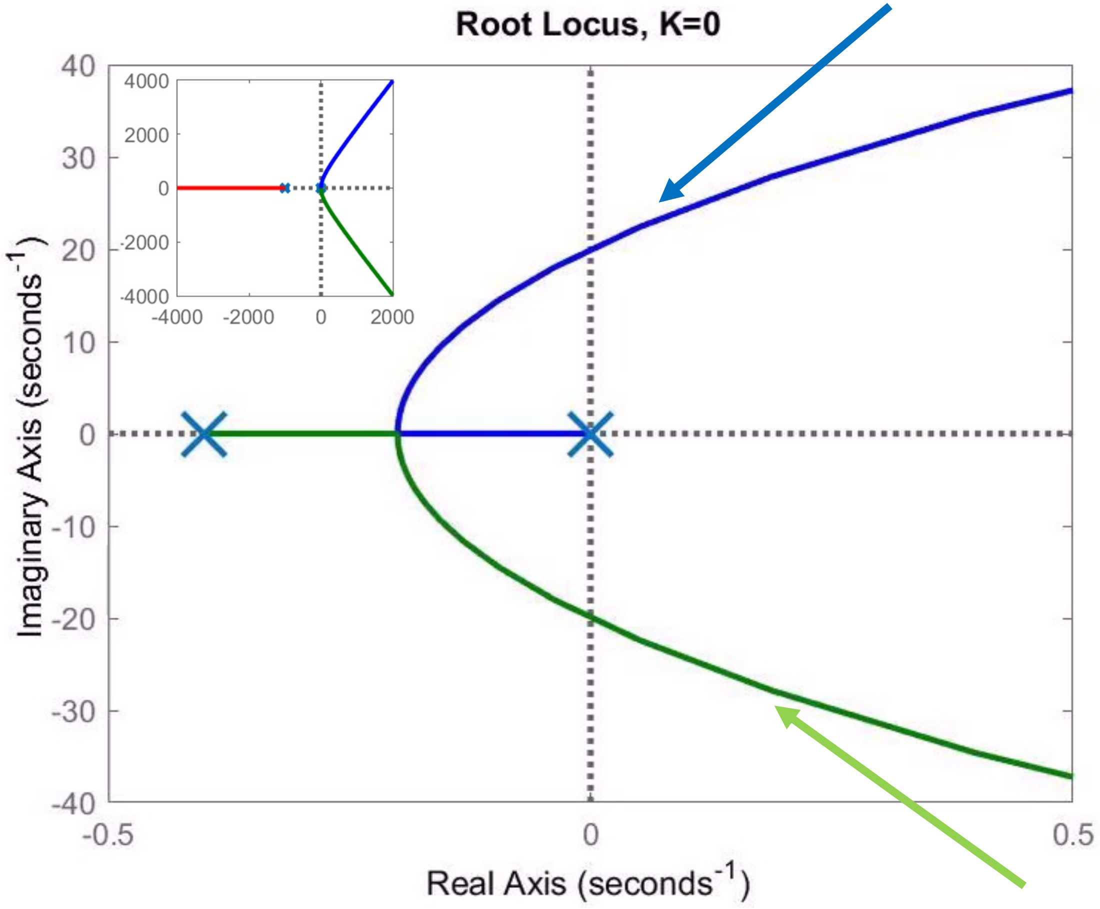
Step response and stability

- ➢ $K < 20$: **asymptotically stable**
  - Dominant poles have negative real parts
  - Oscillation, converging to $1/K$
- ➢ $K = 20$: **Marginal stable**
  - Real part of the dominant poles is 0
  - Undamped oscillation
- ➢ $K > 20$: **unstable**
  - Dominant poles have positive real parts
  - Oscillation with an increasing amplitude



Task 3 - root locus

Trajectory 1: locations of the first dominant pole with a varying K



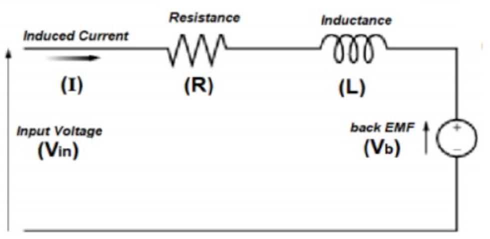
Trajectory 2: locations of the second dominant pole with a varying K

$$G(s) = \frac{10^5}{5s^3 + 5001s^2 + 2000s + 10^5 K}$$

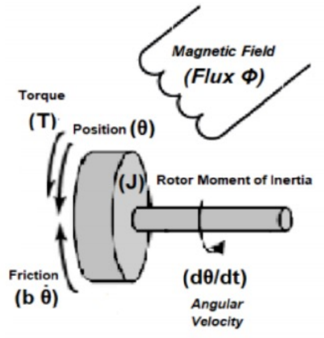
- A *root locus* plot is a graphical representation of the **poles' trajectory** in the complex plane with respect to the **variation of the feedback <u>gain</u> (K).**
- Poles are labelled by x, zeros by o (if there is any)
- If we add poles with the variation of K on root locus plot, poles will overlay on the root locus trajectories!

Task 4 – Performance of an approximated system

DC Motor Equivalent Circuit



DC Motor Free Body Diagram



Can we control the angle speed of a **fan**?

fan →

$$\boldsymbol{G_\omega(s) = \frac{10^5}{5s^2 + 5001s + 2000}}$$

prize wheel →

$$\boldsymbol{G_\theta(s) = \frac{10^5}{5s^3 + 5001^2 + 2000s}}$$

(1) Poles of the fan system

$$G_\omega(s) = \frac{10^5}{5s^2 + 5001s + 2000}$$

Poles $p_{1,2} = \frac{-5001 \pm \sqrt{5001^2} - 4\cdot5\cdot2000}{2\cdot5} \Rightarrow \boldsymbol{p_1 = -0.4}$ and $\boldsymbol{p_2 = -999.8}$

negative real poles
➔ **stable!**

(2) DC gain of the fan system $\boldsymbol{G_\omega(s)}$

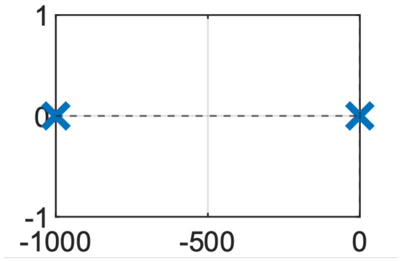
$$\alpha = \lim_{s\to0} s\left(\frac{1}{s}G_\omega(s)\right) = \lim_{s\to0} \frac{10^5}{5s^2 + 5001s + 2000} = \mathbf{50}$$

The angular velocity of the fan will converge to 50 with the unit step input!

Task 4 – Performance of an approximated system

(3)-(4) Dominant pole approximation

Dominant pole $p = 0.4 \Rightarrow$
the approximated system (derived in SG 1)

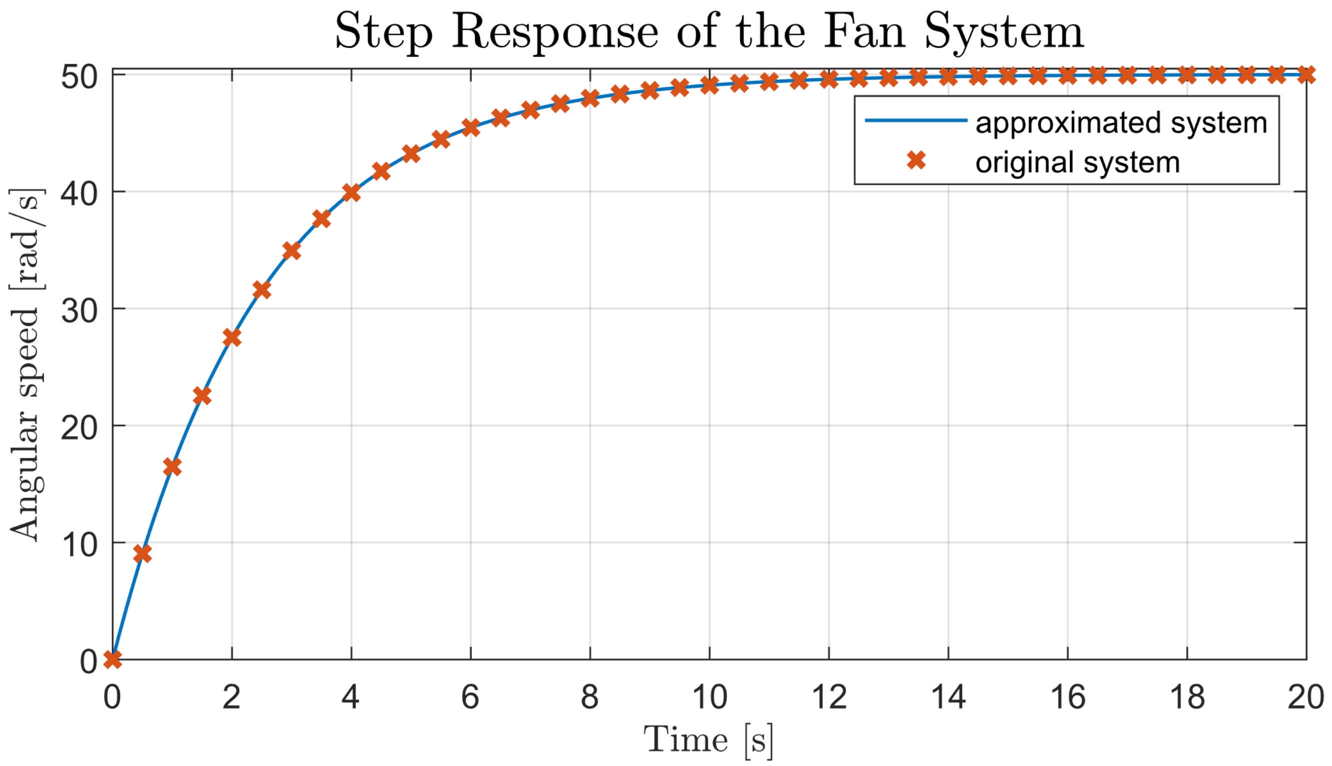


The dominant pole is far away from another pole
➔ good approximation!

$$\widehat{G_\omega}(s) = \frac{K}{s + 0.4}$$

Step response via finite value theorem:

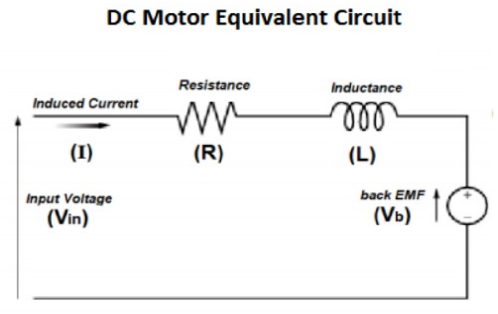
$$\hat{\omega}(t) = 50\left(u(t) - e^{-0.4t}\right)$$

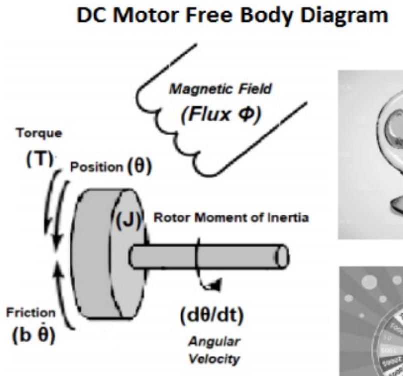


As $t \to \infty$, the step response of the system converges to 50 rad/s.

- ➢ The fan will converge to a fixed wheel angular speed after a period time.
- ➢ We can control the angular speed of the fan!

Task 4 – Performance of an approximated system





Can we control the wheel angle of a **prize wheel**?

fan → $$G_\omega(s) = \frac{10^5}{5s^2 + 5001s + 2000}$$

prize wheel → $$G_\theta(s) = \frac{10^5}{5s^3 + 5001^2 + 2000s}$$

(5) Perform the same analysis to the approximated **prize wheel** system:

The step response does NOT converge.

- As $t \to \infty$, the wheel angle tends to infinite large.
- Therefore, the angle of the prize wheel **cannot** be controlled by a step input.