Imperial College London

BIOE40002 – Signals and ***Control***

MATLAB practical 3 – PID controllers

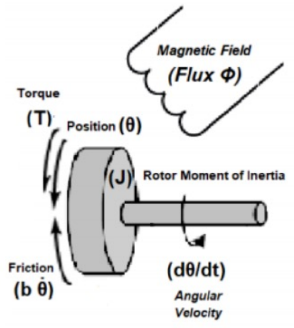
Learning objective

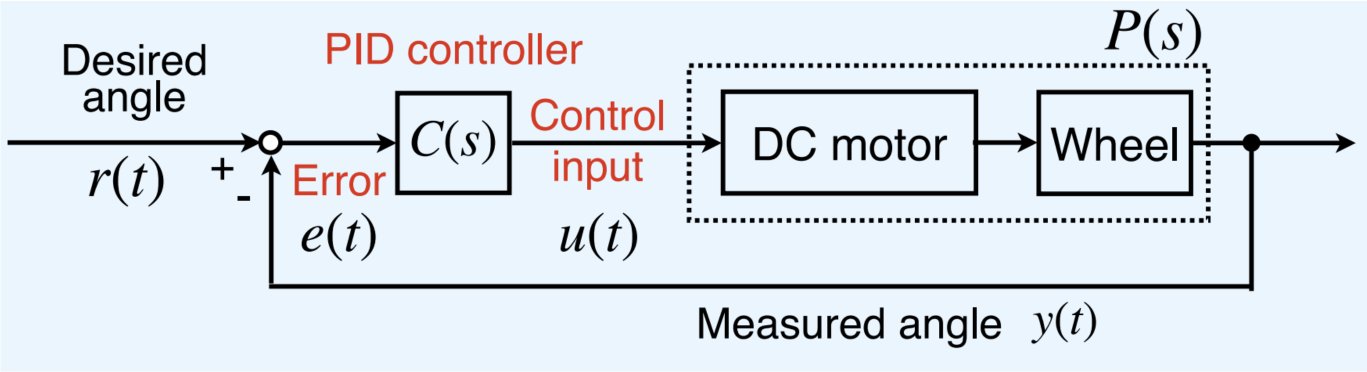
By the end of this MATLAB session you should be able to:

- Explain what the P, I and D in PID control signify.
- Design PID controllers to achieve the desired response of a given system.

PID controllers - recap

DC Motor Free Body Diagram





$$u(t) = K_P e + K_I \int_0^t e(\tau)d\tau + K_D \frac{de(t)}{dt}$$

$$C(s) = K_P + \frac{K_I}{s} + K_D s$$

K_P
Proportional gain

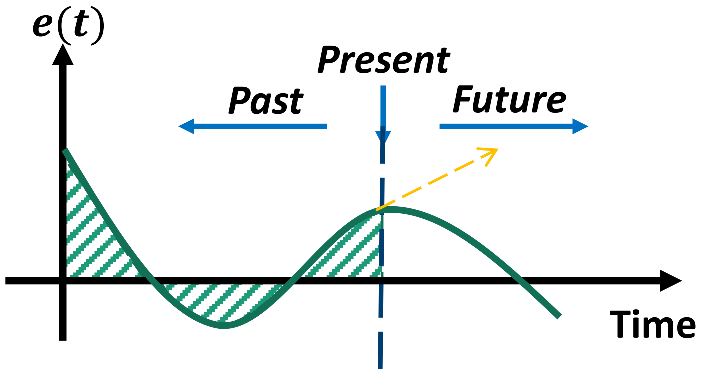
Present

K_I
Integral gain

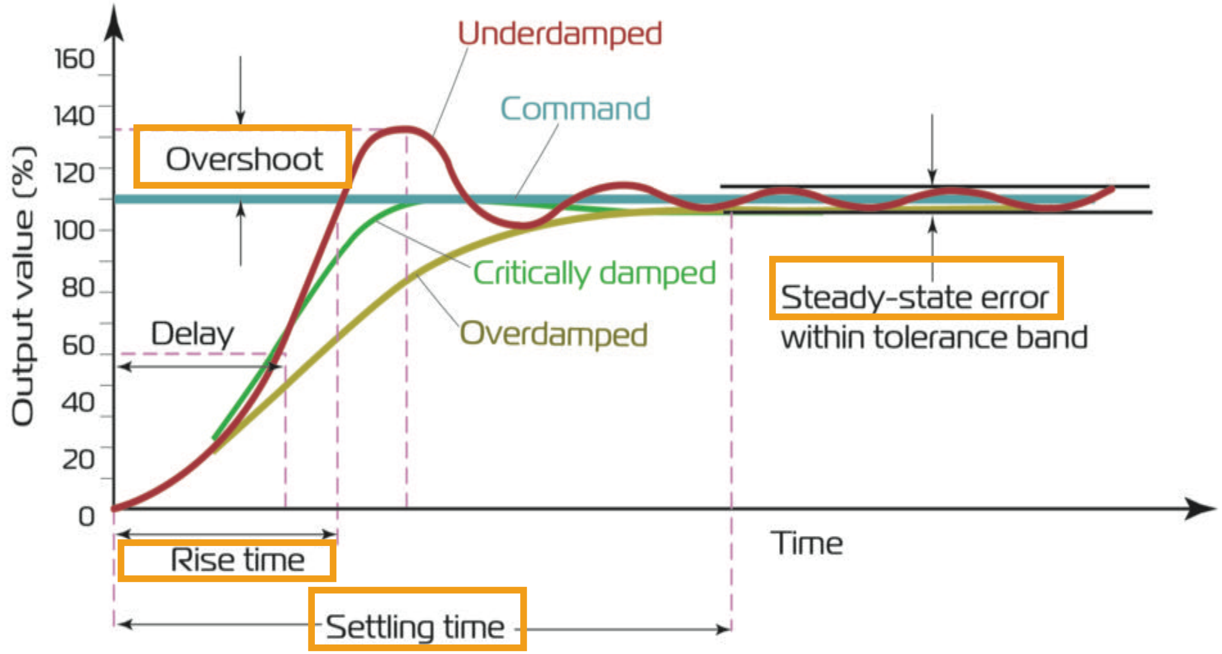
Past

K_D
Derivative gain

Future



System response to K_P, K_D and K_I - recap

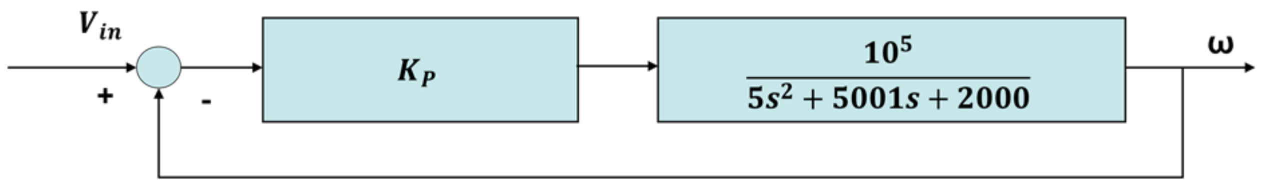


Increase of	Overshoot	Rise Time	Settling Time	Steady-state Error
K_P	Increase	Decrease	Small increase	Decrease
K_I	Increase	Decrease	Increase	Eliminate
K_D	Decrease	Decrease	Decrease	No impact

Today's task

Investigate the effects of K_P, K_I and K_D to the system dynamics

Task 2 – System dynamics of *P* controller



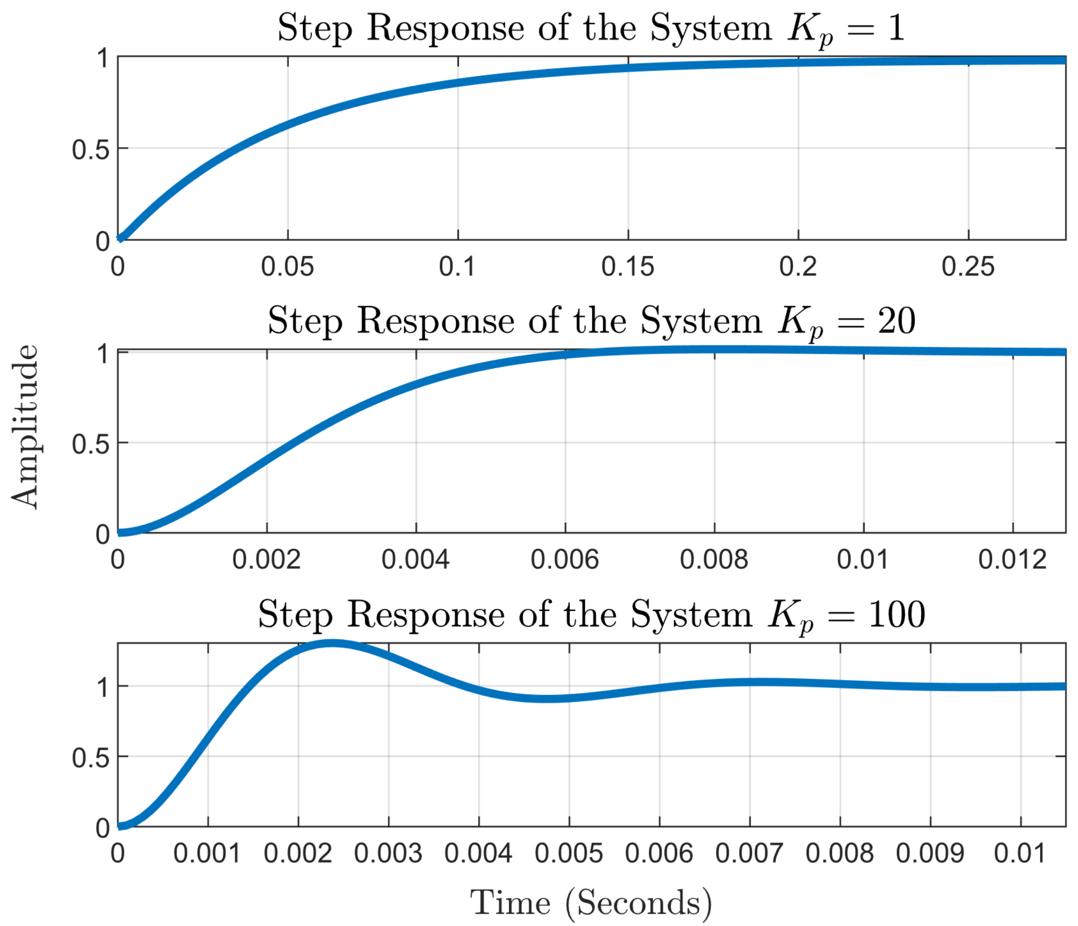
$$G(s) = \frac{10^5 K_P}{5s^2 + 5001s + (2000 + 10^5 K_P)}$$

(2) - (3)

As the proportional gain K_P increases

- the **rise time** decreases, as the step response grew faster
- the **overshoot** appears at $K_P = 20$

K_P	Overshoot (%)	Rise Time (s)
1	0	0.11
20	1.74	0.0038
100	30.5	9.9e-4



Task 2 – System dynamics of ***P*** controller

$$G(s) = \frac{10^5 K_P}{5s^2 + 5001s + (2000 + 10^5 K_P)}$$

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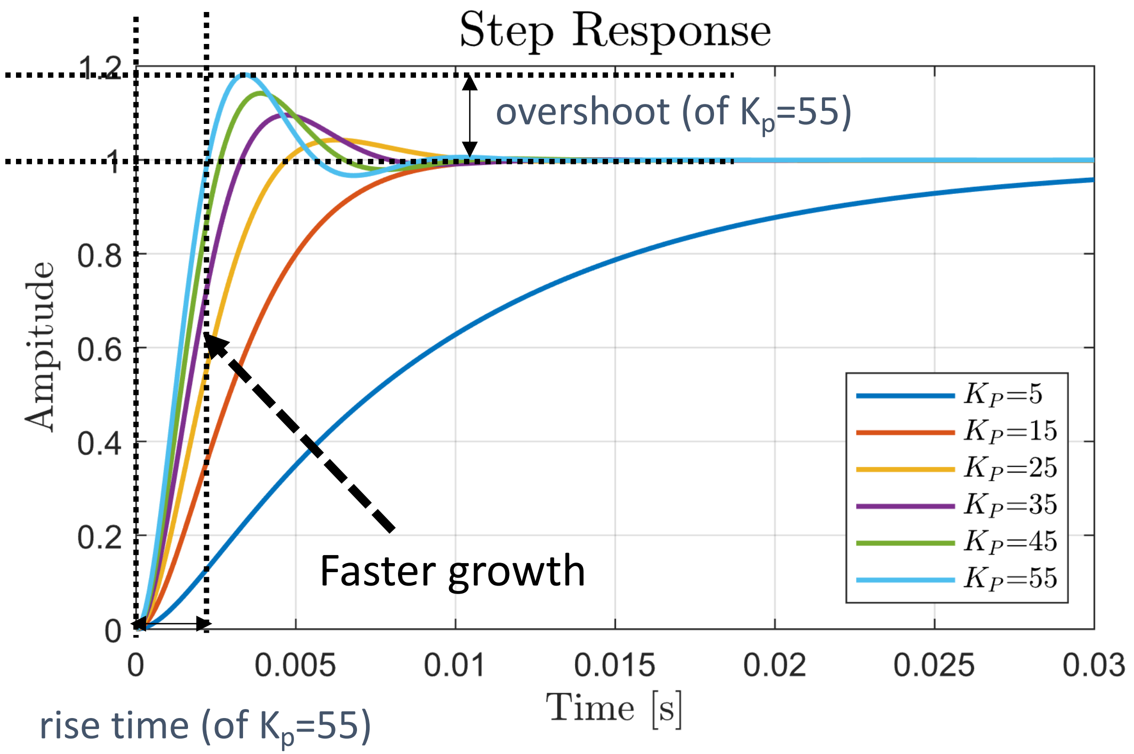
(4)

As the proportional gain K_P increases

- the **rise time** decreases, as the step response grew faster
- the **overshoot** appears at $K_P = 15$ (not very obvious) and increases with K_P.



Task 2 – System dynamics of *P* controller

(4)

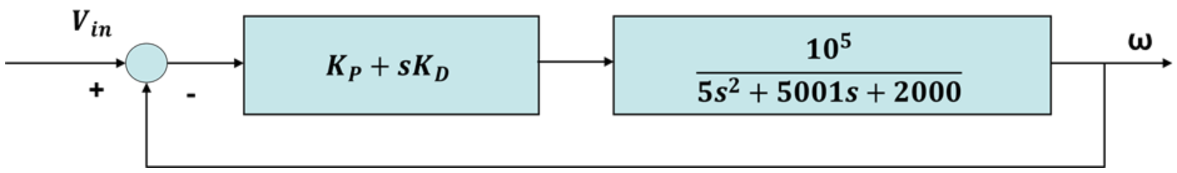
Kp	Overshoot	Settling Time	Steady-State Error [%]
5	0	0.036	0.8
15	0.09	0.009	0.13
25	4.33	0.008	0.08
35	9.62	0.007	0.06
45	14.25	0.008	0.04
55	18.2	0.008	0.04

$$G(s) = \frac{10^5 K_P}{5s^2 + 5001s + (2000 + 10^5 K_P)}$$

```
[y, t] = step(sys)
steady_state_error = abs(1-y(end)).*100
```

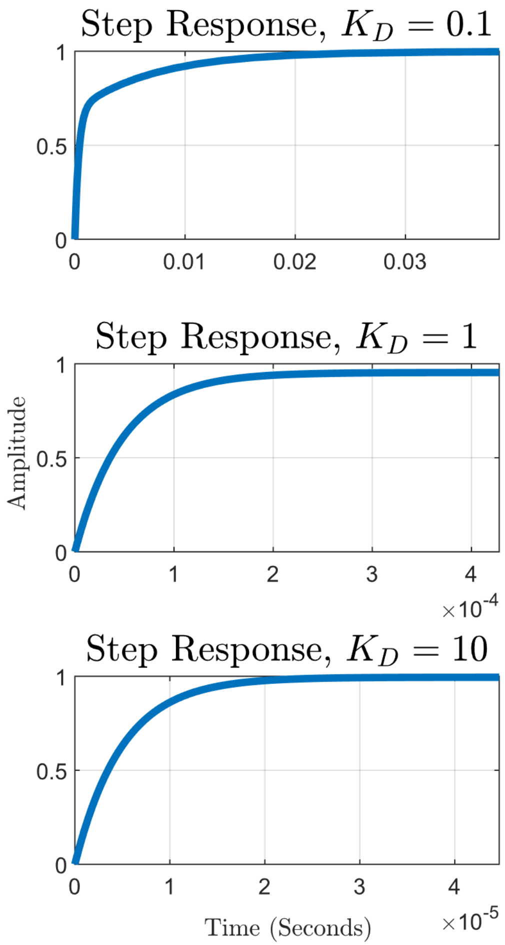
- As K_P increases, the **overshoot** increases, the **steady-state error** decreases.
- It is **not possible** to achieve a small error (<0.1%) and a small overshoot (<1%) simultaneously, because there is a trade-off between the overshoot and steady-state error!

Task 3 – System dynamics of ***PD*** controller



$$G(s) = \frac{10^5(sK_D + K_P)}{5s^2 + (5001 + 10^5 K_D)s + (2000 + 10^5 K_P)}$$

(2)

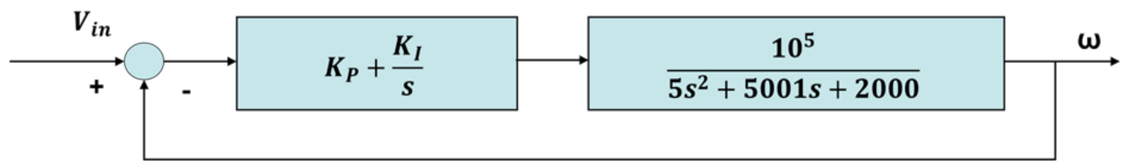


(3)

Kd	Overshoot	Steady-State Error [%]
0.1	0	0.24
1	0	4.66
10	0	0.51

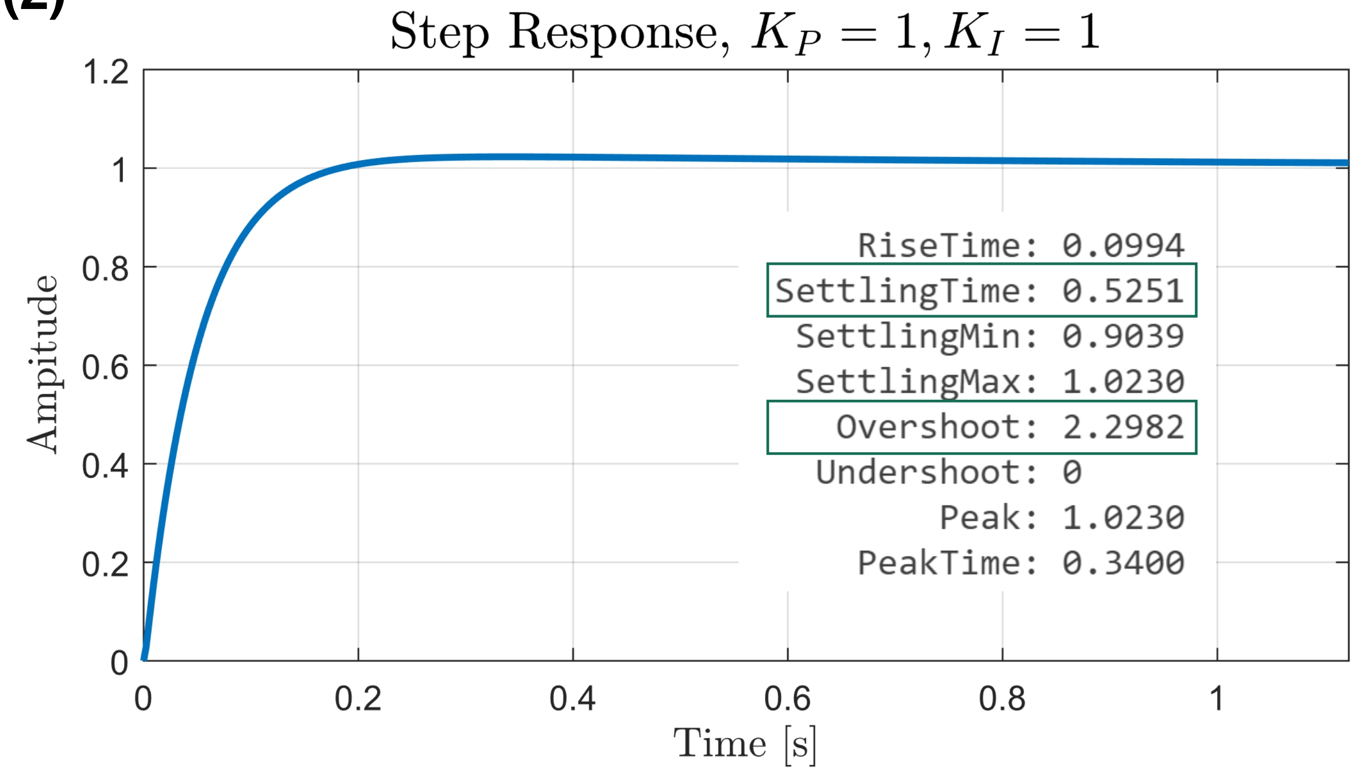
- There is **no overshoot** with D controller.
- **DC gain** is the ratio of output/input in the steady-state.
- By Final Value Theorem, $\text{DC gain} = \frac{1}{\frac{0.02}{K_P} + 1}$. Thus, the level of the **steady-state error** (the difference between the DC gain and the initial input) is determined solely by K_P but independent of K_D.

Task 4 - System dynamics of ***PI*** controller



$$G(s) = \frac{10^5(sK_P + K_I)}{5s^2 + 5001s + (2000 + 10^5 K_P + 10^5 K_I)}$$

(2)



- By Final Value Theorem, DC gain $= 1$, which is independent of the value of K_I.
- DC gain = input ➔The steady-state error can be completely removed!

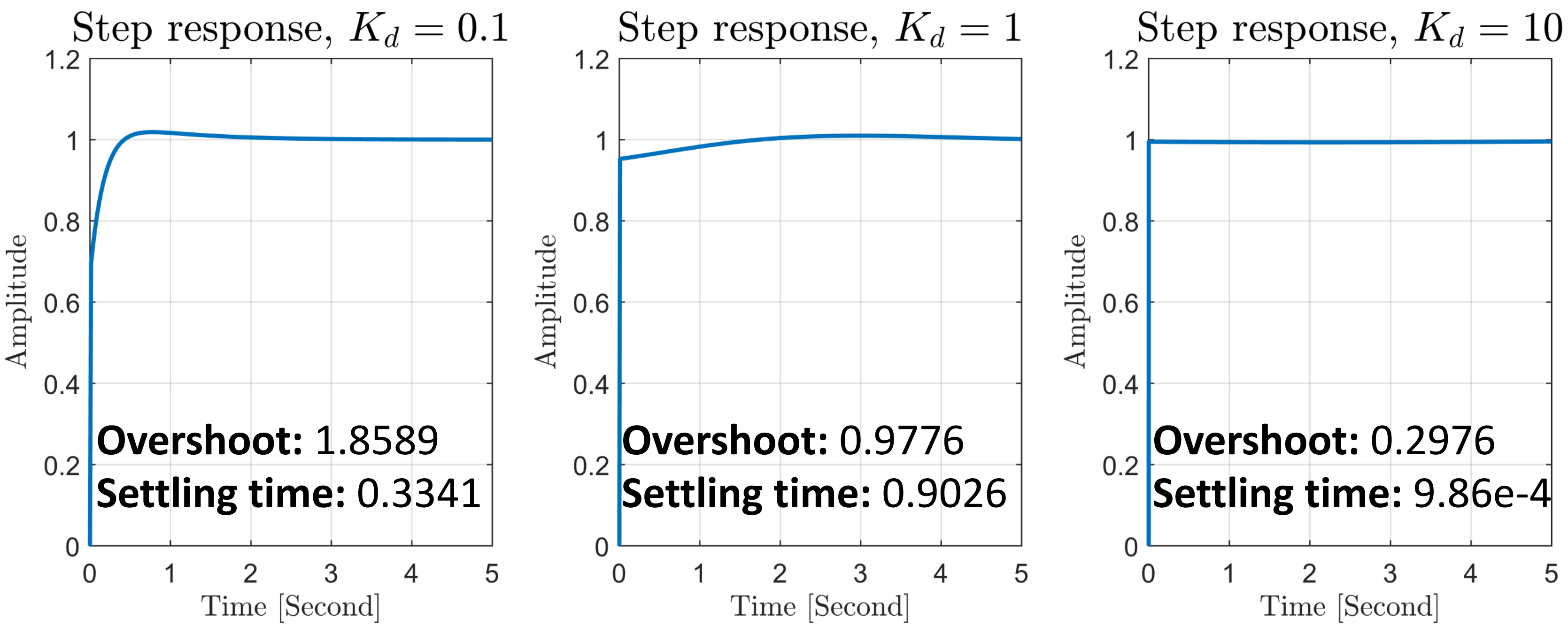
Task 5 - System dynamics of *PID* controller

$$G(s)=\frac{10^5(K_I+sK_P+s^2K_D)}{5s^3+(5001+10^5K_D)s^2+(2000+10^5K_P)s+10^5K_I}$$

$$\begin{cases} K_P = 1 \\ K_I = 1 \end{cases}$$

(1)

- **Step response**



The step response is 'flatter' when $\boldsymbol{K_d = 10}$.

- Both the overshoot and settling time are the smallest
- $K_D = 10$ gives the best performance when $K_P = 1$.