Imperial College London

BIOE50011 – Signals and ***Control***

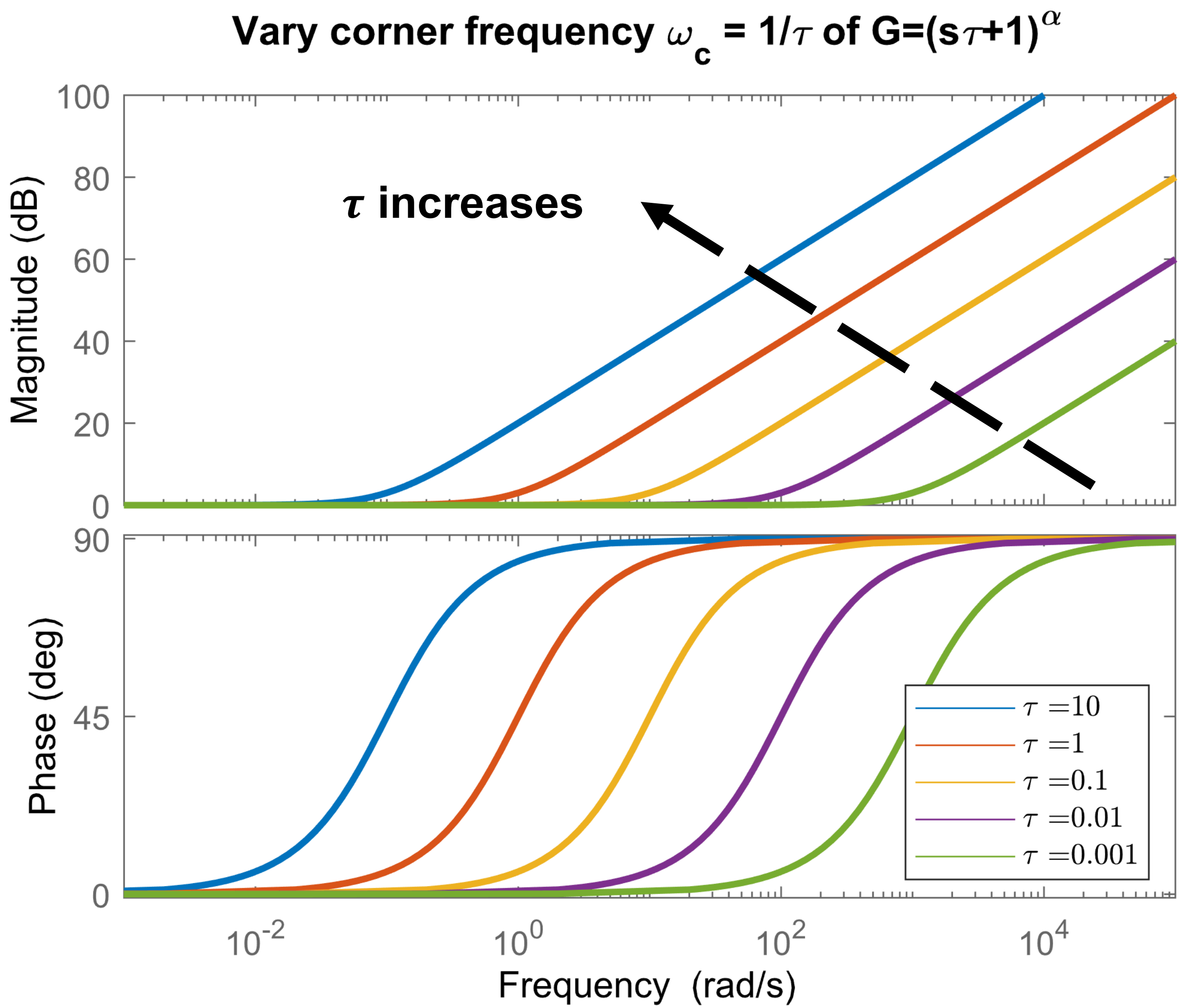
MATLAB practical 4 – Bode plots

Learning objective

By the end of this MATLAB session you should be able to:

- Draw Bode plots for LTI systems.
- Recognize typical Bode plots for 1st and 2nd order systems.
- Identify transfer function from Bode plots.
- Compute and interpret gain and phase margins.

Task 2 - Bode plots of 1st order factor



$$G(s) = (s\tau + 1)^{\alpha}$$

$$\boldsymbol{\alpha = 0, \tau\ [0.001, 0.01, 0.1, 1, 10]}$$

The **cut-off frequency,** $\omega_c = 1/\tau$, is inversely proportional to τ.

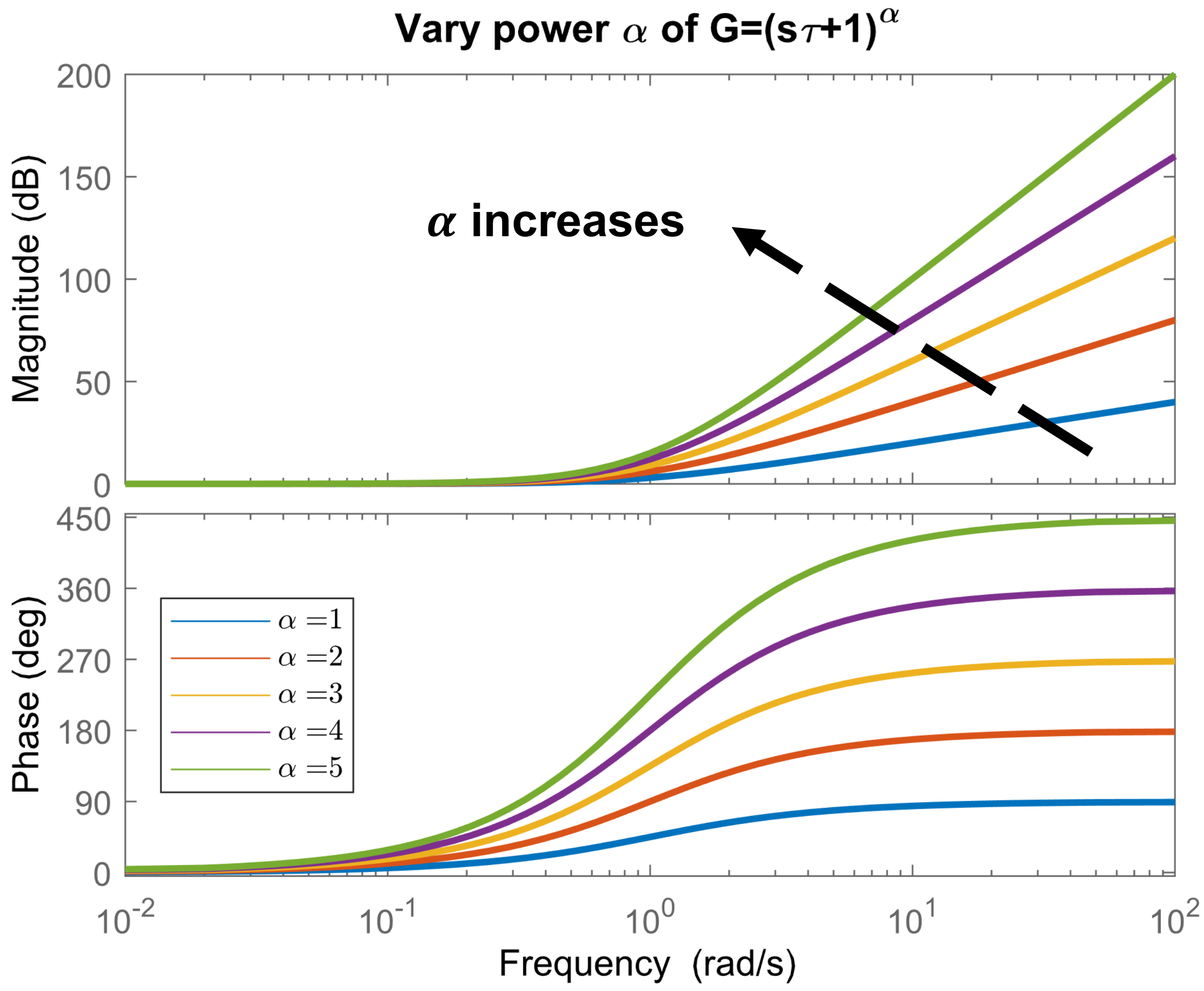
ω_c decreases as τ increases.

Magnitude $\approx 20\log(1) = 0$ for $\omega \ll 1/\tau$

and $\approx 20\log(1) + 20\log(\omega\tau)$

$= 20\log(\omega\tau)$ for $\omega \gg 0$

Task 2 - Bode plots of 1st order factor



$$G(s) = (s\tau + 1)^{\alpha}$$

$$\boldsymbol{\tau = 10 \text{ with } \alpha \in [1, 2, 3, 4, 5]}$$

The corner frequency, ω_c, is fixed and independent of the value of α.

As α increases

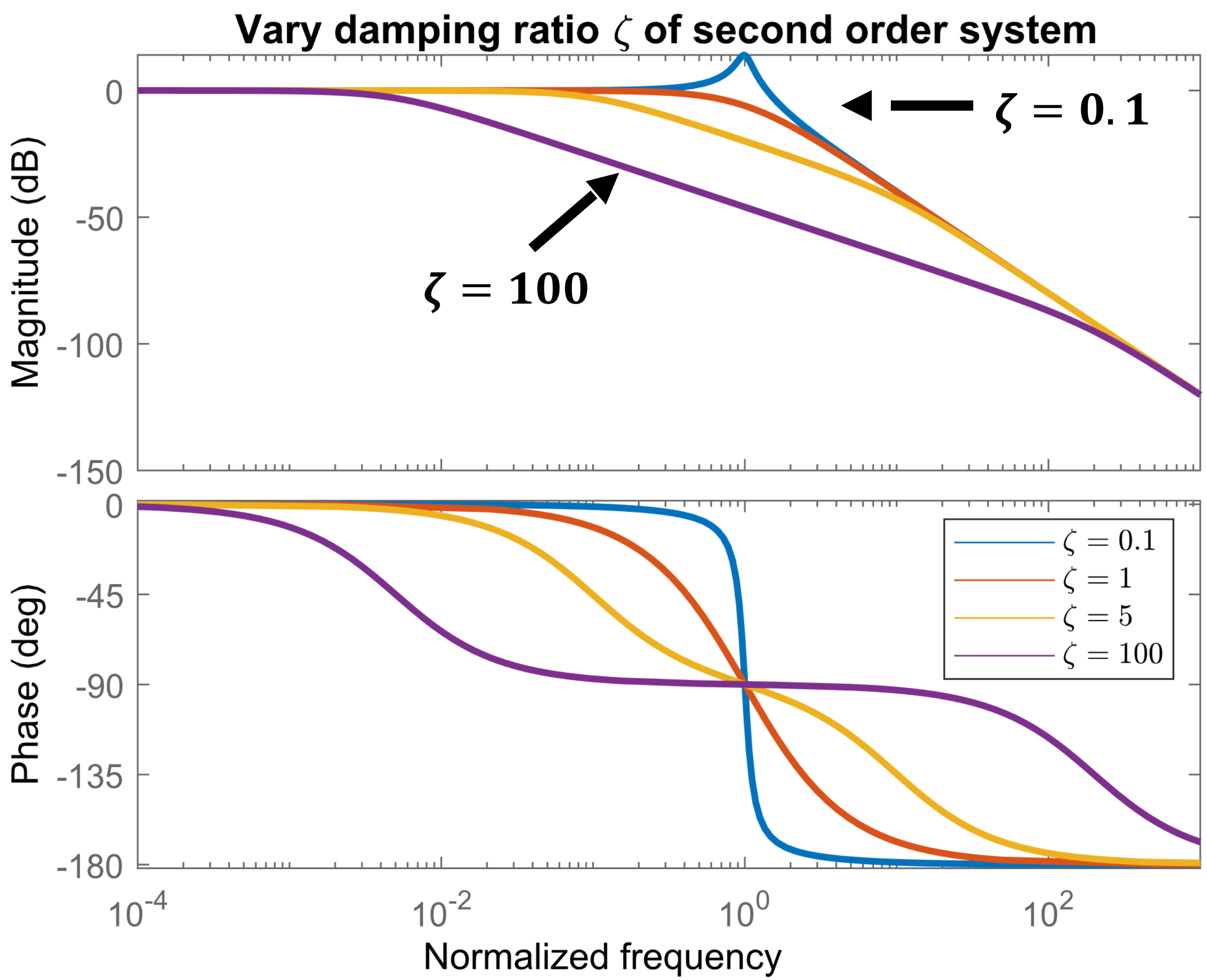
the slope of the magnitude plot increases

$$\textbf{magnitude} = \boldsymbol{20 \log |j\omega\tau + 1|^{\alpha} \approx \alpha \cdot 20 \log \omega\tau}$$

the asymptote of the phase plot increases

$$\textbf{phase} = \boldsymbol{\angle(j\omega\tau + 1)^{\alpha} = \alpha\angle(j\omega\tau + 1) \approx \alpha \cdot 90^{\circ}}$$

Task 2 - Bode plots of 2nd order systems



$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Undamped natural frequency $\boldsymbol{\omega_n = 1}$

Damping ratio $\boldsymbol{\zeta \in [0.1, 1, 5, 100]}$

- The x-axis is normalized frequency

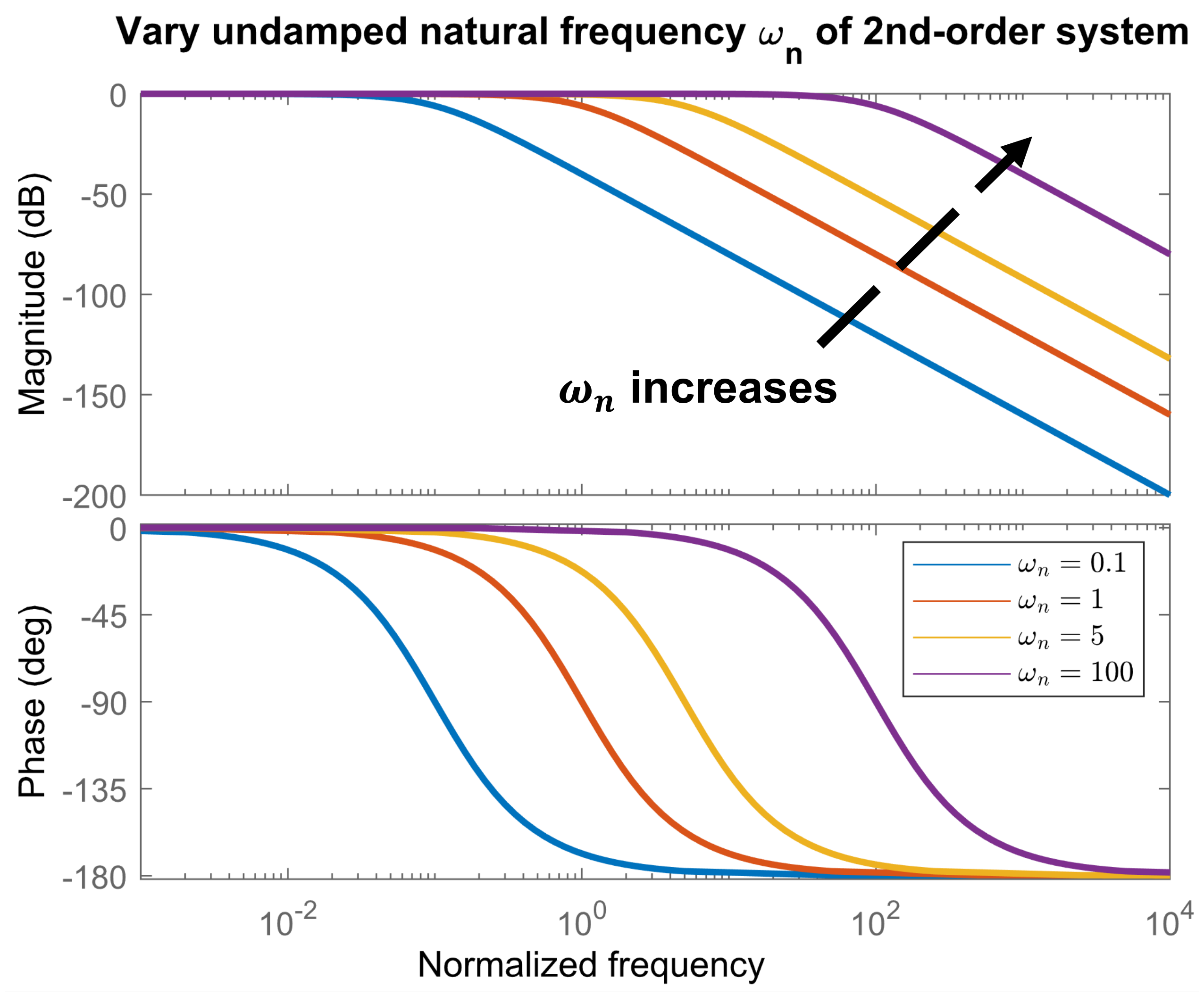
$$r = \omega/\omega_n$$

- $\zeta = 0.1$ creates an overshoot at the resonant frequency:

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2} \approx 0.99 \text{ rad/s}$$

- As ζ increases, a more gradual change in phase is observed around the corner frequency.

Task 2 - Bode plots of 2[nd] order systems



$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Damping ratio $\zeta = 1$

Undamped natural frequency

$\boldsymbol{\omega_n \in [0.1, 1, 5, 100]}$

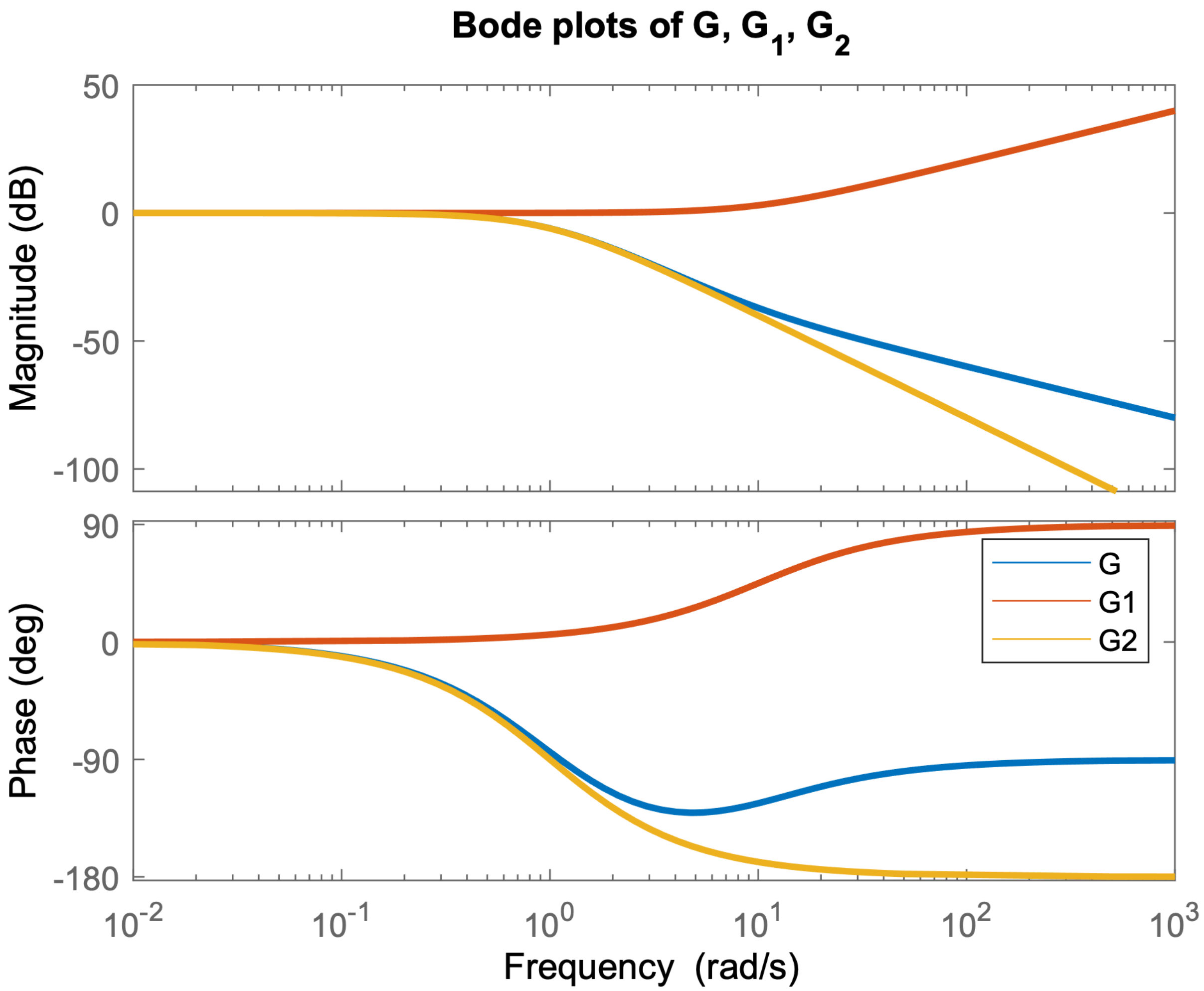
- The x-axis is normalized frequency

$$r = \omega/\omega_n$$

- As ω_n increases,

the corner frequency increases

Task 3 - Bode plots of a product of basic factors



$G_1(s) = 0.1s + 1$

$G_2(s) = (s+1)^{-2}$

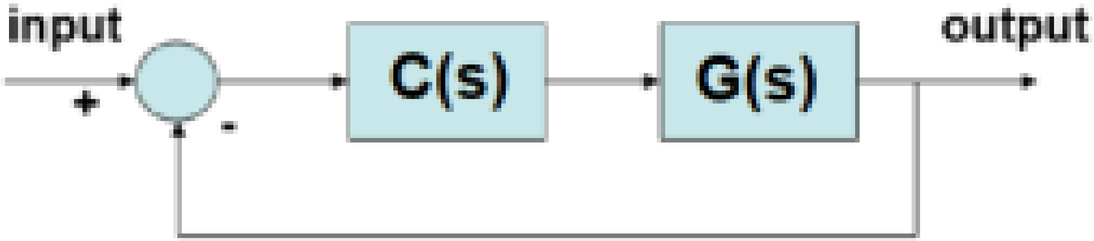
$$G(s) = G_1(s)G_2(s) = \frac{0.1s+1}{(s+1)^2}$$

Bode plots are additive!

- A factor with a positive power makes the magnitude and phase **increase**.
- A factor with a negative power makes the magnitude and phase **decrease**.

Task 4 – Stability margins

The closed-loop system of interest:



$$G(s) = \frac{10^6}{(s+10)(s+1000)} \qquad C(s) = K = 1$$

Transfer function:

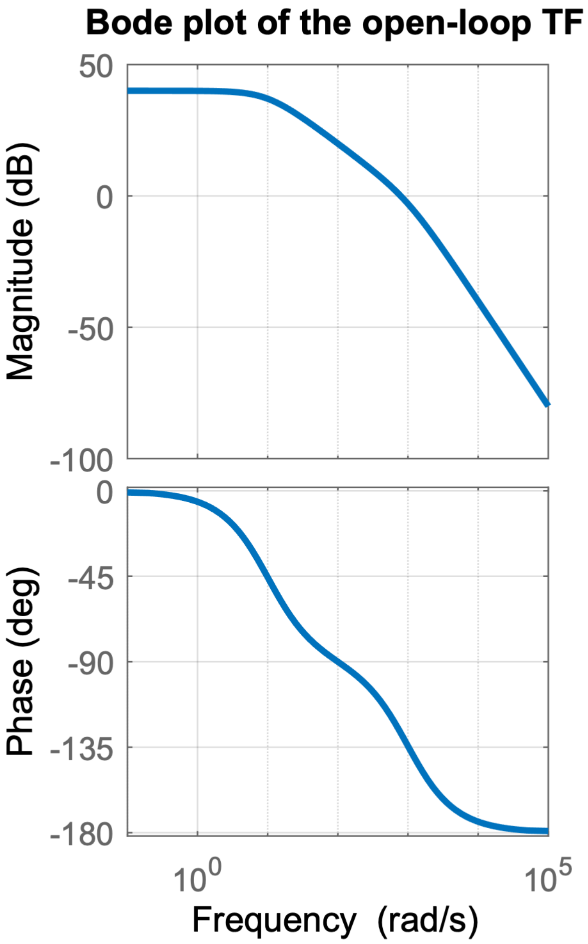
- Open loop

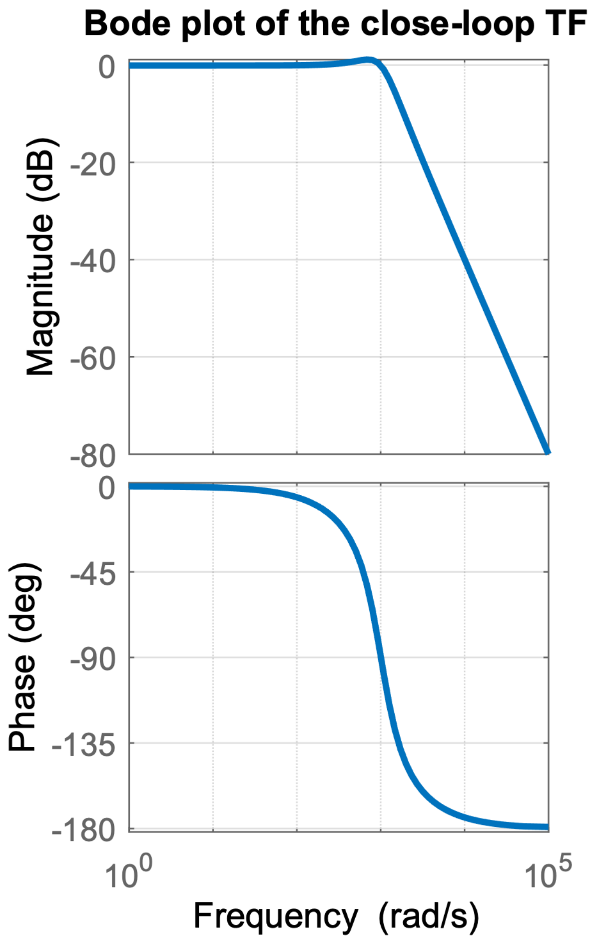
$$G_1(s) = C(s)G(s)$$

- Closed loop

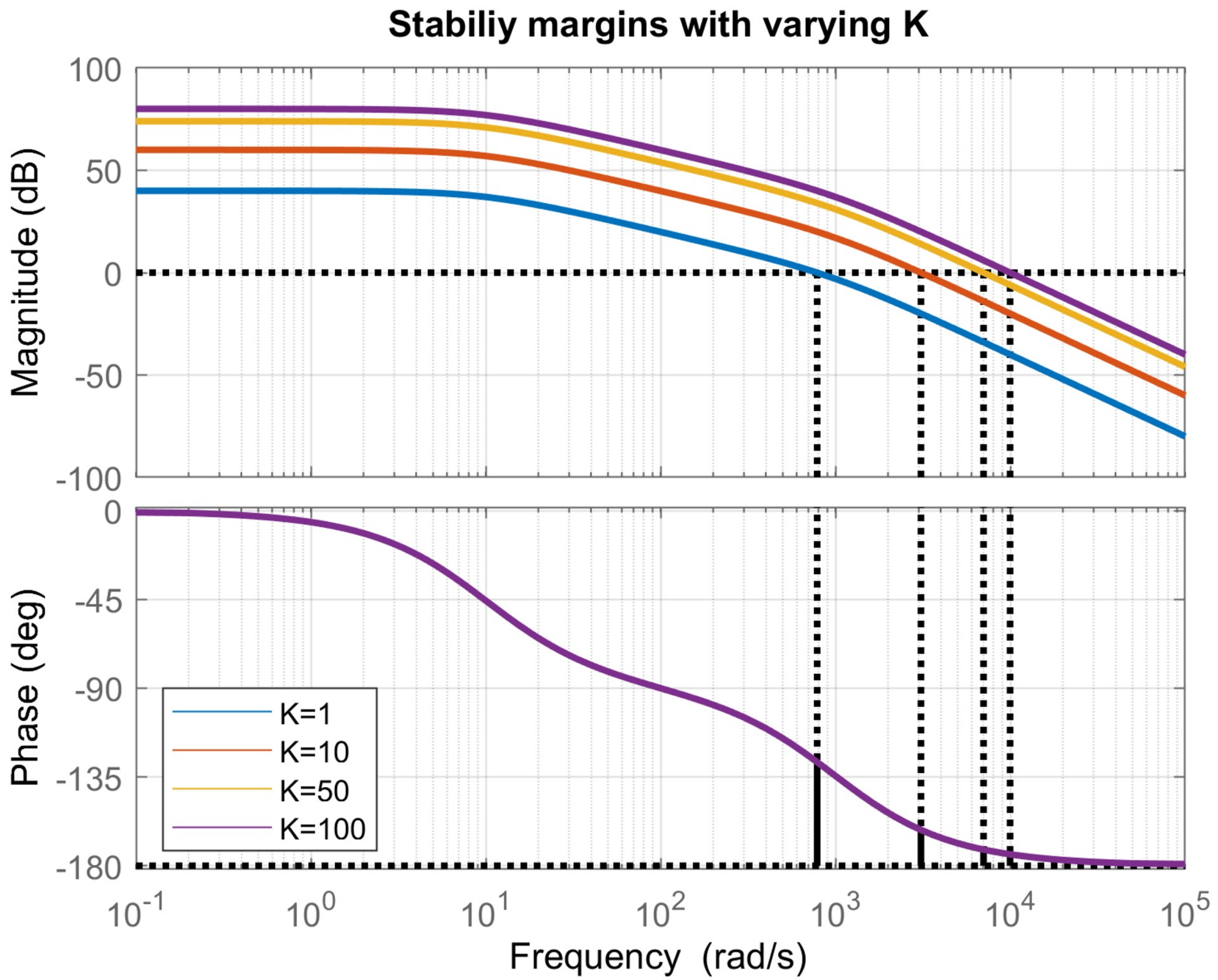
$$G_2(s) = G_1(s)/(1 + G_1(s))$$

Bode plots of the systems:





Task 4 – Stability margins

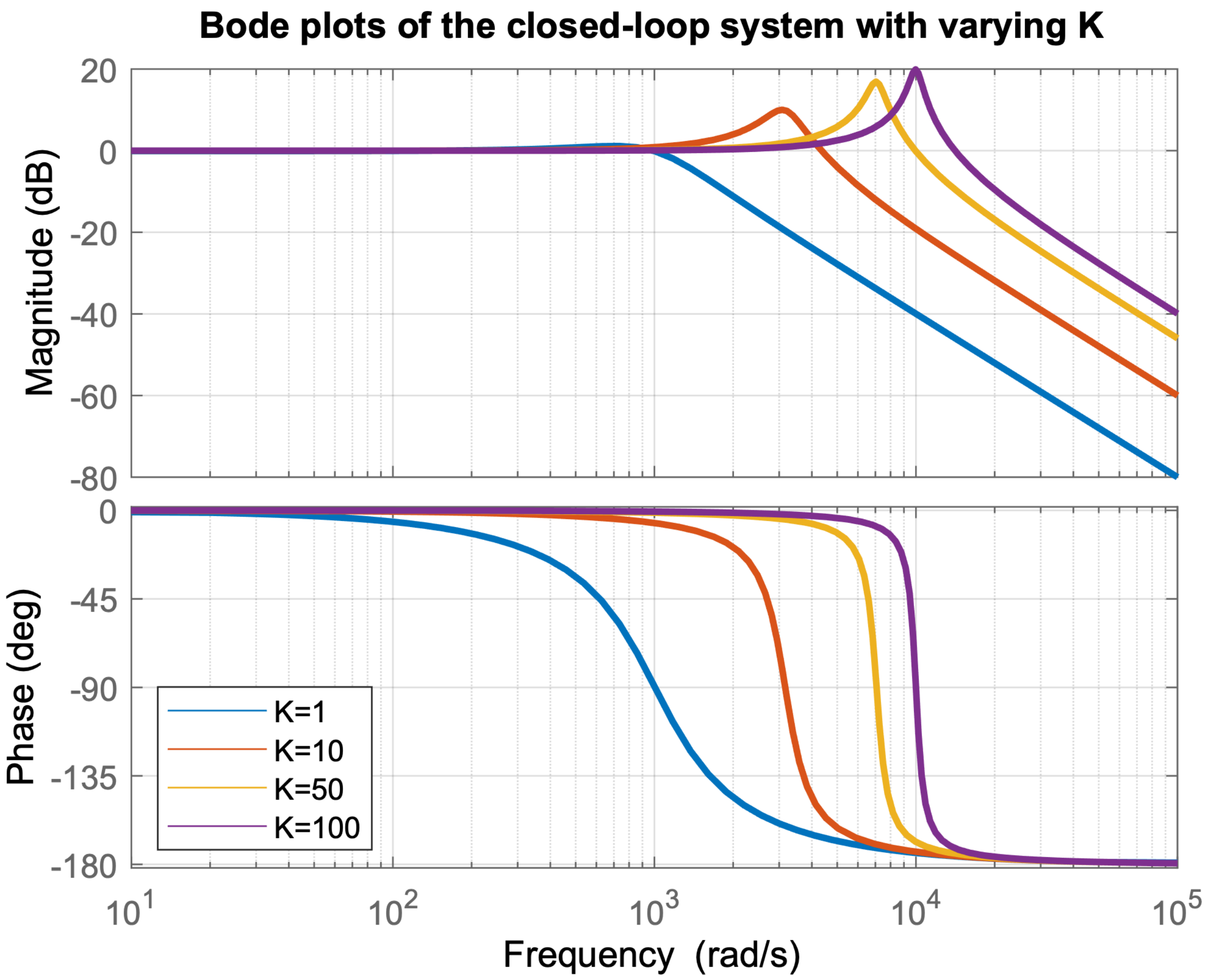


The phase margins of the system decrease as K increases.

K	PM	ω_g (rad/s)
1	52.6	786
10	18.2	3084
50	8.2	7036
100	5.8	9975

The gain margin is infinity (`Inf`), regardless of the value of K.

Task 4 – Stability margins



- Gain and phase margins measure the tolerance of a system to variations, i.e. how stable the system is.
- These quantities are determined by the **open-loop** transfer functions.