# Electrical Engineering II - Formula Summary

## 1 BJT

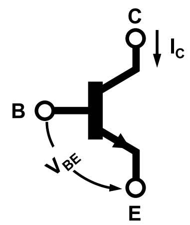



Figure 1: An n-p-n BJT

1. KCL

$$I_E = I_C + I_B$$

2. CCCS

$$\beta = \frac{I_C}{I_B} \qquad I_C = \alpha I_E \; (\alpha = \frac{\beta}{\beta + 1})$$

3. VCCS

$$I_C = I_S \, e^{\frac{V_{BE}}{V_T}}$$

In active mode: for n-p-n BJT, $V_{BE} = 0.7V$, for p-n-p BJT, $V_{EB} = 0.7V$. $V_T \approx 25.6mV$ under room temperature. A modified VCCS with the early voltage is

$$I_C = I_S e^{\frac{V_{BE}}{V_T}} (1 + \frac{V_{CE}}{V_A})$$

with $V_{CE,sat} \approx 0.2V$.

4. Output resistance

$$r_o = \frac{V_A}{I_C} = \frac{\Delta v_{ce}}{\Delta i_c}$$

5. Small signal transconductance

$$g_m = \frac{I_C}{V_T} = \frac{\Delta i_c}{\Delta v_{be}}, \qquad r_e = \frac{1}{g_m}$$

6. Base input resistance

$$r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

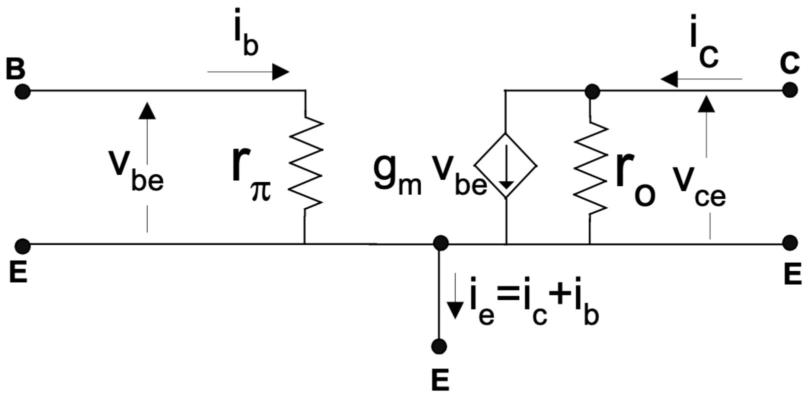



Figure 2: Small signal equivalent for BJT

# 2 MOSFET

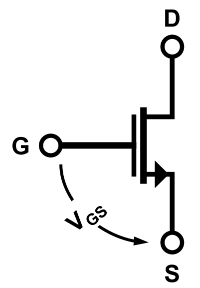



Figure 3: An n-type MOSFET

1. $V_{GS} > V_{TH}, V_{DS} = 0$, no induction current.
2. Triode region $V_{GS} > V_{TH}, V_{DS} < V_{GS} - V_{TH}$
$$I_D = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})V_{DS}$$
3. Onset of saturation $V_{GS} > V_{TH}, V_{DS} = V_{GS} - V_{TH}$
$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})^2$$
4. Saturation $V_{GS} > V_{TH}, V_{DS} > V_{GS} - V_{TH}$[1]
$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})^2$$
with the channel width modulation
$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$$
where $\lambda = \frac{1}{|V_A|}$.
5. Output resistance
$$r_o = \frac{1}{\lambda I_D} = \frac{V_A}{I_D}$$
6. Body effect equation
$$V_{TH} = V_{TH,o} + \gamma[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}]$$
7. Small signal transconductance
$$g_m = \sqrt{2\mu_n C_{ox}}\sqrt{\frac{W}{L}}\sqrt{I_D} = \frac{2I_D}{V_{GS} - V_{TH}}$$

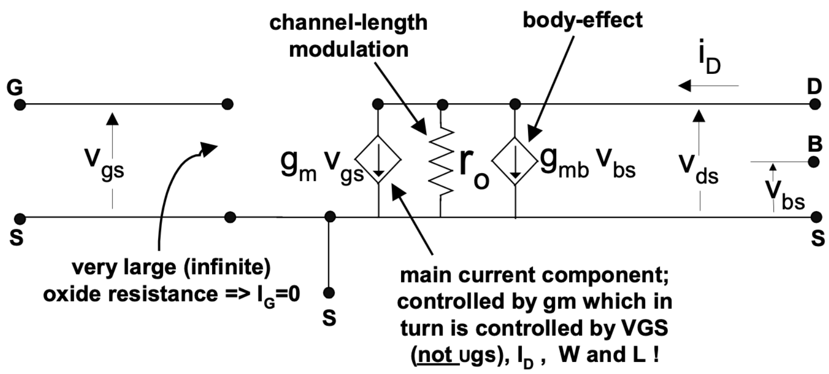



Figure 4: Small signal equivalent for MOS

# 3 Current Mirrors, Current and Voltage References

1. Simple current mirror
$$\frac{I_{out}}{I_{in}} = \frac{1}{1 + 2/\beta} \qquad \frac{I_{out}}{I_{in'}} = \frac{1 + \frac{V_{CE,2}}{V_A}}{1 + \frac{V_{CE,1}}{V_A}}$$
2. $\beta$-compensated current mirror
$$\frac{I_{out}}{I_{in}} = \frac{1}{1 + 2/\beta^2}$$

[1]bizarrely, $V_{DS}$ is also documented as $v_{DS}$, but they are the same.

3. MOS simple current mirror

$$\frac{I_{out}}{I_{in}} = \frac{(\frac{W}{L})_{out}[1+\lambda v_{DS,out}]}{(\frac{W}{L})_{in}[1+\lambda v_{DS,in}]}$$

4. $V_{BE}$-based ref

$$I_{ref} = \frac{V_{EB}}{R}$$

5. $V_T$-based ref

$$I_{ref} = \frac{V_T}{R}\ln(\mu)$$

6. $V_{BE}$ multiplier

$$V_{out} = kV_{BE}\frac{(\frac{W}{L})_5}{(\frac{W}{L})_2}$$

7. $V_T$ multiplier

$$V_{out} = kV_T\ln(\mu)$$

8. Bandgap reference

$$\frac{\partial V_{EB}}{\partial T} = -2.2mV/°C \qquad \frac{\partial V_T}{\partial T} = 0.086mV/°C$$
$$V_{out} = 25.6V_T + V_{BE} \qquad V_{BG} = 1.21 - 1.31V$$

# 4 Amplifiers

| | Common Emitter | Common Emitter with Emitter Resistance |
|---|---|---|
| $A_V(\frac{V_0}{V_i})$ | $-\frac{R_C}{r_e}$ | $\approx -\frac{R_C}{R_E + r_e}$ |
| $A_{VS}(\frac{V_0}{V_S})$ | $\frac{R_b \parallel R_i}{R_S + (R_b \parallel R_i)}A_V$ | $\frac{R_b \parallel R_i}{R_S + (R_b \parallel R_i)}A_V$ |
| $R_i$ | $\approx \beta r_e$ | $\approx \beta(R_E + r_e)$ |
| $R_{out}$ | $R_C$ | $R_C$ |

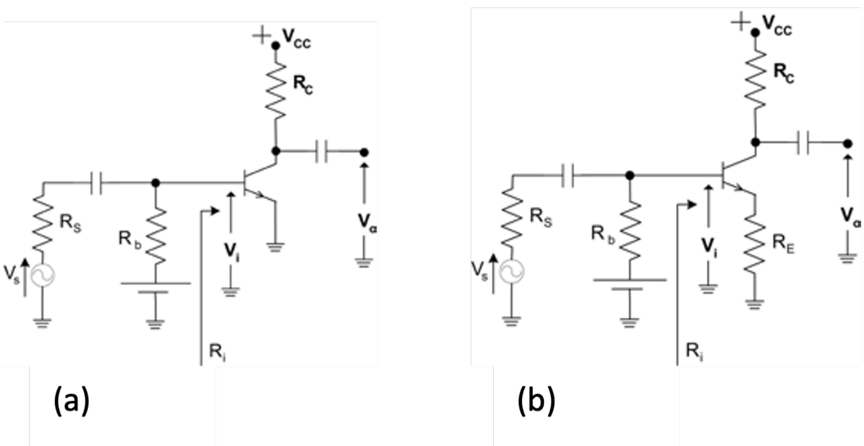



Figure 5: (a) Common emitter, (b) Common emitter with emitter resistance

- DCLL

$$I_C = -\frac{V_{CE}}{R_C} + \frac{V_{CC}}{R_C}$$

- ACLL

$$i_c = -\frac{V_{CE}}{R_C \parallel R_L} + \frac{V_{CC}}{R_C \parallel R_L} - \frac{R_C}{R_L}I_C$$

- Optimal, $Q$

$$I_{CQ^*} = \frac{1}{R_C \parallel R_L} \frac{R_L}{R_C + 2R_L} V_{CC}$$

$$V_{CEQ^*} = \frac{R_L}{R_C + 2R_L} V_{CC}$$

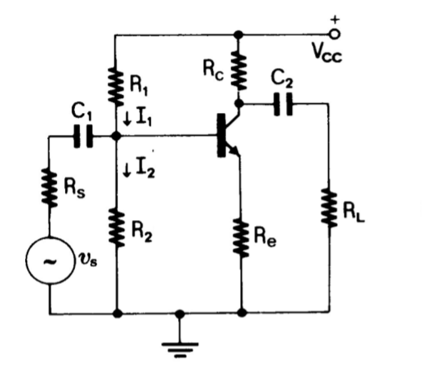
- Intersection of ACLL and biasing line

$$V_{CE} = V_{CC} - R_C I_{CQ}$$

- Emitter degenerated ACLL

$$i_c = -\frac{V_{CE}}{R_e + (R_C \parallel R_L)} + \frac{V_{CC}}{R_e + (R_C \parallel R_L)} - \frac{R_C \parallel R_L}{R_e + (R_C \parallel R_L)} \frac{R_C}{R_L} I_C$$

$$I_{CQ} = \frac{1}{R_e + (R_c \parallel R_L)} V_{CEQ} \qquad V_{CEQ} = \frac{R_L}{R_C + 2R_L} V_{CC}$$




- Emitter degenerated DCLL

$$I_C = -\frac{V_{CE}}{R_C + R_E} + \frac{V_{CC}}{R_C + R_E}$$