Electromagnetics - Formula Summary

1. Coulomb's Law:

$$\vec{F} = \frac{q_1\, q_2}{4\,\pi\,\varepsilon_r\,\varepsilon_0\, r^2}$$

2. Electric Field:

$$\vec{E} = \frac{\vec{F}}{q}$$

Field of a point charge	$E = \frac{kQ}{r^2}$
Field inside a capacitor	$E = \frac{V}{d}$
Superposition	$\vec{E}_{net} = \sum_{i=1}^{N} \vec{E}_i$
Electric Flux	$\Phi_E = \int \vec{E} \cdot \mathrm{d}\vec{A}$

3. Electrical Potential:

$$V = \frac{U}{q}$$

$$\Delta V = V_f - V_i = -\int_i^f \vec{E}\,\mathrm{d}\vec{l}$$

4. Gauss's Law:

$$\oint \vec{E} \cdot \mathrm{d}\vec{A} = \frac{Q_{enc}}{\varepsilon}, \qquad \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

$$\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0, \qquad \nabla \cdot \vec{B} = 0$$

5. Ampere's Law:

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu \int (\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}) \cdot \mathrm{d}\vec{A}$$

$$\nabla \times \vec{B} = \mu(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t})$$

6. Faraday's Law:

$$\oint \vec{E} \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \vec{B} \cdot \mathrm{d}\vec{A}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

7. Magnetic Field:

- A straight line wire

$$B = \frac{\mu_0\, I}{2\,\pi\, r}$$

- Flux:

$$\Phi_B = \int \vec{B} \cdot \mathrm{d}\vec{A}$$

8. Biot-Savart Law:

$$\vec{B} = \oint \frac{\mu\, I\, \mathrm{d}\vec{l} \times \hat{r}}{4\pi r^2}$$

9. Ohm's Law:

$$\vec{J} = \sigma \vec{E}$$

10. Inductance:

$$L = \frac{\Phi_T}{I} = \frac{\mu_0 N^2 A}{l}, \qquad \varepsilon = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

11. Capacitance:

$$C = \frac{Q}{V}$$

12. Poyuting Vector:

$$\vec{S} = \vec{E} \times \vec{H} \quad \text{where } \vec{H} = \frac{\vec{B}}{\mu_0}$$

13. Total Energy Density:

$$E_d = \frac{1}{2}E^2\varepsilon + \frac{1}{2}\mu H^2$$

14. Electromagnetics Waves:

- Speed of wave

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

- Refractive index

$$n = \frac{c}{v} = \sqrt{\mu_r \varepsilon_r}$$

- Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \qquad \sin\theta_c = \frac{n_2}{n_1}$$

- Boundary continuity

	dielectric-dielectric	dielectric-PEC ($\vec{E}_2 = 0$)
$\parallel$	$E_{t,1} = E_{t,2}$, $H_{t,1} = H_{t,2}$	$E_{t,1} = 0$
$\perp$	$\varepsilon_1\, \hat{\mathbf{n}} \cdot \vec{E}_1 - \varepsilon_2\, \hat{\mathbf{n}} \cdot \vec{E}_2 = \rho_s$ $\hat{\mathbf{n}} \cdot \vec{H}_1 = \hat{\mathbf{n}} \cdot \vec{H}_2$	$\varepsilon_1\, \hat{\mathbf{n}} \cdot \vec{E}_1 = \rho_s$ $\hat{\mathbf{n}} \cdot \vec{H}_1 = 0$, $\hat{\mathbf{n}} \times \vec{H}_1 = \vec{J}_s$

- Reflection coefficient, Transmission coefficient

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \qquad T = \frac{2\,\eta_2}{\eta_2 + \eta_1}$$

$$\text{where } \eta = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu_0}{\varepsilon}}$$

$$\Rightarrow \quad T = 1 + \Gamma$$

- Parallel polarization:

– Incident wave

$$\vec{E}_i = E_0\, e^{j(\omega t + k_1(x\sin\theta_i + y\cos\theta_i))} (-\cos\theta_i\,\hat{\mathbf{x}} + \sin\theta_i\,\hat{\mathbf{y}})$$

$$\vec{H}_i = \frac{E_0}{\eta_1}\, e^{j(\omega t + k_1(x\sin\theta_i + y\cos\theta_i))}(-\hat{\mathbf{z}})$$

– Transmitted wave

$$\vec{E}_t = TE_0\, e^{j(\omega t + k_2(x\sin\theta_t + y\cos\theta_t))} (-\cos\theta_t\,\hat{\mathbf{x}} + \sin\theta_t\,\hat{\mathbf{y}})$$

$$\vec{H}_t = \frac{TE_0}{\eta_2}\, e^{j(\omega t + k_2(x\sin\theta_t + y\cos\theta_t))}(-\hat{\mathbf{z}})$$

– Reflected wave

$$\vec{E}_r = \Gamma E_0\, e^{j(\omega t + k_1(x\sin\theta_r - y\cos\theta_r))} (-\cos\theta_r\,\hat{\mathbf{x}} - \sin\theta_r\,\hat{\mathbf{y}})$$

$$\vec{H}_r = \frac{\Gamma E_0}{\eta_1} e^{j(\omega t + k_1(x\sin\theta_r - y\cos\theta_r))}\,\hat{\mathbf{z}}$$

- Fresnel Equations

$$T_{\parallel} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$T_{\perp} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\Gamma_{\perp} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

- Brewster angle

$$\sin\theta_i = \frac{1}{\sqrt{1 + \varepsilon_1/\varepsilon_2}}$$

Constants

- Charge on electron: $e = 1.60 \times 10^{-19}$ C
- Permittivity of free space: $\varepsilon_0 = 8.85 \times 10^{-12}$ $\mathrm{C^2/Nm^2}$
- Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ Tm/A
- $k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$ $\mathrm{Nm^2/C^2}$
- Light of speed: $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3.0 \times 10^8$ m/s